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A COMPUTATIONAL ANALYSIS OF THE EFFECTIVENESS OF FAST FOURIER TRANSFORM IN DENOISING ECG SIGNALS

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Abstract. The acquisition of biological signals like ECG includes the presence of noise from power line interference, muscle artefacts etc. Mathematical functions like Fourier Transform (FT) can be used to selectively eliminate specific noise frequencies from the signals. FT converts 1-D signals in their time-domain to frequency-domain. Subsequently, setting the amplitude of the noise frequency to zero, and applying the inverse FT function reconverts the signal back to its time-domain. This process removes the noise from the signals. In this paper we computationally demonstrate denoising the ECG signals using Fast Fourier Transform algorithm.

1. Introduction

Fourier transform (FT) is a mathematical technique, which transforms a function in the time -domain to a function in frequency domain. To go back from frequency domain to time domain, we can use inverse Fourier transform. The idea of Fourier Transform is derived from the Fourier series. The Fourier series is the representation of any periodic function in terms of trigonometric or exponential function.
Consider a periodic function $f(x)$ in the interval $-L \leq x \leq L$.
The Fourier Trigonometric Series representation of the function is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n cos\left(\frac{2n\pi x}{L}\right) + b_n sin\left(\frac{2n\pi x}{L}\right) \tag{1.1}$$

where a_0, a_n and b_n are the Fourier coefficients given by
$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx, a_n = \frac{1}{L}\int_{-L}^{L} f(x)cos\left(\frac{n\pi x}{L}\right)dx$ and
$b_n = \frac{1}{L}\int_{-L}^{L} f(x)sin\left(\frac{n\pi x}{L}\right)dx$ where $n = 1, 2, ...$ The Fourier Exponential series equivalent to equation 1.1 is

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in2\pi x}{L}} \ where \ c_n = \frac{1}{L}\int_{-L}^{L} f(x)e^{-\frac{in2\pi x}{L}}dx \tag{1.2}$$

This particular expression in equation 1.2 will help us to extend the Fourier Series to Fourier Transform. The concept of Fourier Transform extends the Fourier series

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to non periodic functions. This can be achieved by taking the period $L \to \infty$.(i.e., for any non-periodic signal, we can say its time period is infinity). As $L \to \infty$, the angular frequency $\omega_0 = \frac{2\pi}{L}$ becomes extremely small and $n\omega_0$ can take any value. Define a new variable $\omega = n\omega_0$ and take $F(\omega) = Lc_n$.
The Fourier transform is $F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx$
Likewise, the inverse Fourier Transform is $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{-i\omega x}dx$
The Discrete Fourier Transform (DFT) deals with finite discrete signals and finite or discrete number of frequencies. The Fast Fourier Transform (FFT) is an optimized algorithm for the implementation of the Discrete Fourier Transformation. It determines the DFT of an input significantly faster than computing it directly. One can refer [1, 2] for more details regarding Fourier series and Fourier transforms.
Fourier Transform has wide range of applications in signal analysis and processing, wireless communications, image processing etc. It helps to identify and remove noise from the signals by converting a signal from its time domain to frequency domain, where the noise frequency can be specifically removed. The process of transforming signals to frequency domain and further denoising signals have found application for the Fourier Transform principles in biological sciences. The FT algorithms are used for the identification and characterization of biomolecules, diagnosis of tumors, processing of biological signals like electrocardiogram (ECG), electroencephalogram (EEG) etc. For more applications of Fourier Transform, refer [3].
In this research work, we performed the denoising of ECG signals computationally employing FFT algorithm and determined the effectiveness of this process. ECG signals are quasi-periodic signals with a standard frequency range of 0.5 to 150 Hz and an amplitude of 0.02 – 5 mV. ECG waves represent the electrical activities i.e., the depolarization and repolarization events in the cardiac tissues during each cardiac cycle [4]. In the time domain, a typical ECG wave is composed of P wave, a QRS complex (composed of Q wave, R wave and S wave), and T wave. ECG signals are useful in assessing the cardiovascular health. Alterations in the characteristic of the component waves of ECG indicate cardiovascular disorders like arrhythmia. Therefore, efficient methods of recording ECG is essential for diagnostic purposes.
ECG signals can get corrupted by several different noises including power line interference, baseline wander, muscle artefacts and instrumentation noises [5]. Therefore, it is essential to have high quality methods to capture the signal, which have low noise interference, and efficient signal processing methods to obtain clean signals that can be used in a clinical setup. Fourier Transform is one of the mathematical principles that can be applied to denoise an ECG signal. In this work, we have employed the FFT algorithm to denoise a few selected ECG signals from the MIT-BIH Arrhythmia database and assessed the fidelity of the processed signal.

2. Methods

The ECG signals used in the study were accessed from the MIT-BIH Arrhythmia Database [6] available on PhysioNet [7]. The MIT-BIH database is a collections of two-channel ambulatory ECG recordings (sampling rate 360 Hz, 11-bit resolution over a 10 mV range) of 47 subjects studied by the Boston's Beth Israel Hospital (BIH) Arrhythmia Laboratory between 1975 and 1979. For this study, we have randomly selected five raw signals from the records- 100, 105, 118, 220 and 234, without any pre-processing. Each of these records are 30 minutes long, and only the signal from the modified limb II (MLII) was used for the analyses.
The analyses and visualization of data was performed using Python (3.12 stable release). The Python library- Waveform Database Software Package (WFDB) [8], was used to access and process the MIT-BIH ECG signals from PhysioNet. The other Python libraries used for data processing include- Librosa (for audio analysis) [9], os, numpy (for numerical computing) [10], pandas (for data analysis and manipulation) [11], matplotlib (for visualization) [12], SciPy (for scientific computing) [13].
The signal processing method using Fourier Transform performed in this study is illustrated in **Figure 1**. All the selected records of ECG signals were subjected to noise processing in a similar manner.

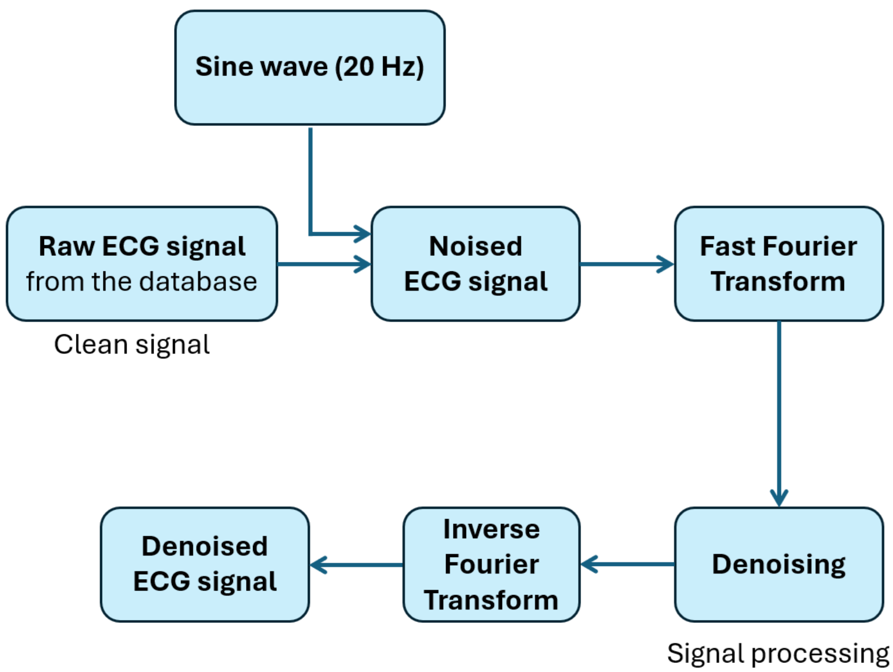

FIGURE 1. An illustration of noising and denoising of ECG signals using FFT.

A. Adding noise to the ECG signal:
The raw ECG signals available in the database were already processed and denoised. Hence, in our preliminary analysis using FFT plots, no noise frequencies were visible. Therefore, to introduce noise in the ECG waves, a sinewave of 20 Hz with the same sampling rate as that of the ECG wave (360/s) and same time duration (30 min), was added to it.
B. Converting the signal from time-domain to frequency-domain:
The *scipy.fft.rfft()* function from the SciPy library was used to compute the discrete Fourier Transform (DFT) of the real-valued ECG signal array, by means of the Fast Fourier Transform (FFT) algorithm. The *rfft()* function takes the normalized noised signal as the input, calculates the FFT of the real sequence and outputs the complex FFT coefficients. The FFT sample frequency points (on x-axis) are plotted against the FFT coefficient values (on y-axis) to obtain the visualization of the transform.
C. Denoising the signal and reconversion back to time-domain:
From the Fourier transform plot, the noise-frequency component (the peak at 20 Hz) was identified and its amplitude was set to zero. Then the signal in the frequency-domain was converted back to its time-domain using the inverse FFT function. The function *scipy.fft.rfft()* computes the inverse of the 1-dimension n-point discrete Fourier Transform of real input computed by *rfft()*. The output time-domain ECG signal is denoised with the noise-frequency (20 Hz) removed.
To assess the fidelity of the denoised signal, we calculated the correlation coefficient (ρ) of the signal with the corresponding raw ECG signal obtained from the database, using the *numpy.corrcoef()* function. The *corrcoef(x,y)* takes the the denoised- and raw ECG signals (in the form of arrays), as input parameters (x and y), and calculates the Pearson product-moment correlation coefficient. The Pearson product-moment correlation coefficient is a measure of the strength of a linear association between two variables. Its value can range from -1 to +1, with -1 exhibiting a negative correlation, +1 showing a positive correlation and 0 representing no relationship.
Figure 2 shows the raw ECG signals of the records 100, 105, 118, 220 and 234 obtained from the MIT-BIH arrhythmia database. Figure 2a shows the plot of the entire 30 min long ECG signal of record 100 [approximately 650000 sampling points at 360 samples/s], whereas the remaining plots (2b-2f) show the ECG signals of the first 1000 sampling points (at 360/s) of all the records. Since the ECG recordings are from subjects having different types of arrhythmias, characteristic abnormalities in the individual wave components (P, Q, R, S, T waves) can be observed.

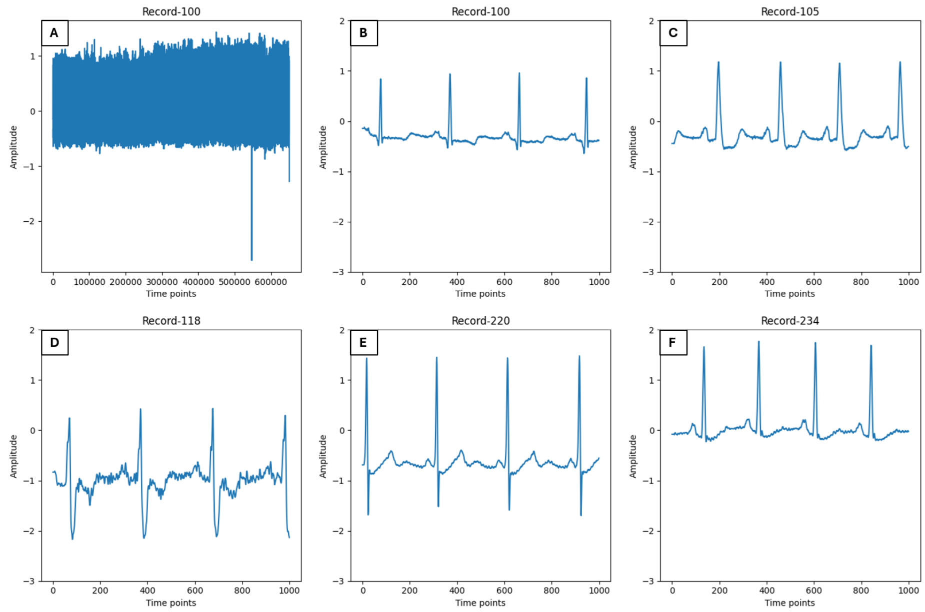

FIGURE 2. The five ECG signals accessed from the MIT-BIH Arrhythmia Database. Each signal has a sampling rate of 360/s. (A) Record 100- entire ECG signal of 30 minutes (approx. 650000 sampling points); The remaining subplots show the signal in the initial 1000 sampling points: (B) Record 100; (C) Record 105; (D) Record 118; (E) Record 220; (F) Record 234.

The process of denoising an ECG signal using Fast Fourier Transform algorithm using the *rfft()* function of the SciPy library is shown in **Figure 3**. The steps include adding a 20 Hz wave as a noise (3b) to the raw ECG signal (3a), to obtain the noise-mixed signal (3c). Subsequent Fourier transform of the signal (3d) converts it into the frequency domain. Deleting the noise component (3e) and applying the inverse Fourier transform using the irfft() function returns the denoised signal back to the time domain (3f).
Since the visual comparison of the the raw signal (3a) and the denoised ECG signal (3f) reveals slight variations in the signal patterns in the latter, the signals were analyzed to determine their correlation coefficient (ρ). Pearson product-moment correlation coefficient was determined using the NumPy function *corrcoef()*. Table 1 shows the correlation coefficient value of each denoised signal.

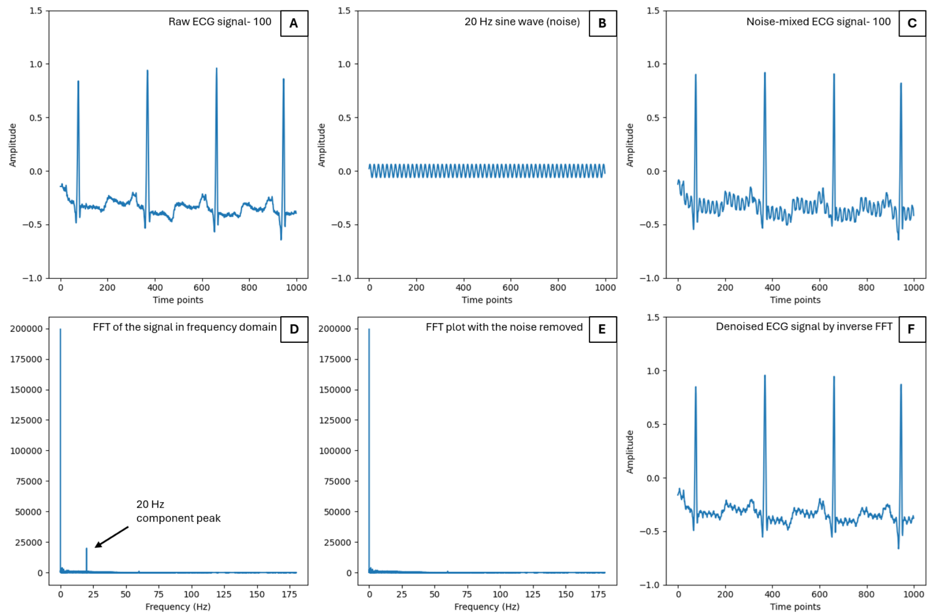

FIGURE 3. The Fast Fourier Transform algorithm based denoising of ECG signal of the record 100. (A) The Raw ECG signal in time domain, showing the initial 1000 sampling points; (B) 20 Hz sine wave as the noise-wave, showing 1000 sampling points at 360 samples/sec; (C) The ECG signal mixed with the noise-wave; (D) Fast Fourier Transform of the noised signal in frequency domain. The marked peak indicates the frequency component of the noise at 20 Hz; (E) The Fourier transform plot with the noise-frequency (20 Hz) amplitude set to zero; (F) The denoised ECG signal in time domain, obtained by the inverse Fourier transform.

Record	Pearson correlation coefficient (ρ)
100	0.9599
105	0.9541
118	0.8148
220	0.9456
234	0.9692

Table 1. Pearson correlation coefficient ρ of the signals. ρ is calculated to assess the linear relationship between the processed denoised ECG signal and its raw signal.

It is observed that a good linear relationship exists between the raw signal and the processed denoised signal. In four out of five records (records- 100, 105, 220, 234), the ρ values showed approximately 95% linearity. In one ECG signal (record-118), ρ was 81.5%. In addition, visual inspection revealed that the denoised waves retained all the features of the ECG signal components (P, Q, R, S, T waves, R-R intervals, QT elevation, gap between the signals etc.) of their raw signal counterparts which were directly accessed from the database.

3. Discussion and conclusion

Fourier transform is a powerful mathematical tool to convert time-domain signals into their frequency domain. The frequency components of the FFT plot offer several advantages, including insights on the different frequencies that constitute the signals, processing and denoising the signals etc. The FT algorithms have wide applications in the field of communication, music industry, analytical and biological signal processing. In this paper we have demonstrated the Fourier Transform processing of ECG signals, in removing noise from the waves.
In every wave signal that is processed, either using the hardware or the software (as we have employed here), it is imperative that the output signal has high fidelity with its input wave. If the process of transformation alters the wave characteristics important features of the signals will be lost. This is particularly important in biological signals like the ECG, where the signals are composed of several components. For example, an ECG wave is composed of P, Q, R, S, T waves. The features of these components, the duration of each component, the time lag between them, their intensity etc. are used for diagnosis of cardiovascular disorders and prognosis of interventions. Therefore, fidelity of the processed wave is an important factor that can affect ECG analyses and interpretation.
Four out of five records showed a very high linearity between the raw signal and the processed signals. However, the method of denoising the signal using FFT algorithm showed a decreased linearity in record 118. Even though the ρ value of 0.815 denotes a good linearity, in our experimental context, that value is considered unsatisfactory due to the following reasons. Firstly, the signals available from the MIT-BIH arrhythmia database are relatively clean and already preprocessed to remove any noise. It can be observed that the FFT plot of the signal (figure 3d, 3e) does not reveal any noise frequency component apart from the one that was introduced by us. Such clean raw signals are seldom obtained using a recording device. Secondly, we have introduced only a single noise-frequency (of 20 Hz) in the ECG signal. In a real-world scenario, while recording a biological signal, noise at multiple frequencies will also be added to the captured signal, such as the line noise from the system, noise from artifacts, etc. This will cause distortion of the signal characteristics. And, to denoise the signal, several frequency bands will have to be removed or suppressed. Such techniques should not alter the wave characteristics required for feature extraction needed for clinical diagnosis. In this experiment, removal a single noise-frequency from a relatively clean signal caused a reduction in the ρ value by 0.15 (equivalent to 15%) in one out of five signals. This implies that additional hardware-based bandpass filters and efficient mathematical algorithms like wavelet transforms should be complemented with Fourier Transform for denoising biological signals.

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