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MODELLING A COMMUNICATION SYSTEM USING STOCHASTIC PETRI NETS WITH FUZZY PARAMETER

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Abstract. In this paper we purpose a Stochastic Petri Net based approach for analysis of communication system with fuzzy sets. There are two stages to the method that is being given. In terms of transition firing rates, the first stage is identical to that of regular stochastic petri nets with steady-state probability and continuous time Markov chain (CTMC) generated. Fuzzy steady state probabilities are computed in the second step after the transition firing rates are represented by triangular fuzzy numbers. A numerical example of communication system considered to analyse the pertinence of proposed method. The significance of this method is to study the dimensions of uncertainty and stochastic variability in system modelling.

1. Introduction

The Introduction of Stochastic Petri Net, [12] and generalized stochastic Petri net (GSPN) by [1] as a high level representations of Markov Chains, contributed a major result in methodological performance evaluation in the 1980's. [2] presented a hierarchical modelling approach that combines queuing network models and GSPNs for the solution of complex models of system behaviour Stochastic Petri Net models were proposed by researchers active in the applied stochastic modelling field, with the goal of developing a tool which allowed in the integration of formal description. The transition firing time is usually described by a Probabilistic distribution and commonly exponential distributions are used. Stochastic petri net acts as a simple and effective method of analysing any system.

The Organization of the paper is as follows, In Section 2, a literature review on PN's is given. In Section 3, the formal definition of stochastic Fuzzy PN's is explained. In Section 4, the presented approach is given in detail. In Section 5, a numerical representation of presented approach for a Communication Protocol is given.

2. Fuzziness in Petri Nets

Fuzzy-time Petri Nets, which are based on the fuzzy enabling duration with the transition of Petri Nets, were first introduced by Valleto, Courvoisier, and Mayeux in 1989. According to [11], the time Petri Nets are defined as the fuzzy intervals with transitions that are associated with the enabling duration. [14] suggested

Key words and phrases. Stochastic Petri Nets, Fuzzy sets, Fuzzy Probability, Continuous – time Markov Chain (CTMC), Communication Systems.

fuzzy-timing high-level Petri Nets for simulating communication protocols. Based on the possibility theory, he developed fuzzy theoretic functions of time, including fuzzy time stamp, fuzzy enabling time, fuzzy occurrence time, and fuzzy delay. There was use of triangular possibility distributions. Fuzzy timed Petri Nets are a novel variation of fuzzy Petri Nets that were proposed by Pedrycz and Camargo (2003). The time factor at the place and transition levels is incorporated into the net structure, which impacts the net performance in terms of input and output place marking distribution at the marking level as well as transition firing. A fuzzy timed Petri Net model was provided by [23], as well as [24] in which the reachability state graph provides the basis for the model's performance analysis.

3. Stochastic Petri Nets

A Petri Net is a bipartite directed graph represented by a quadruple Petri Net $= (P, T, P_r, P_o)$ where $P = \{p_1, p_2, p_3, \ldots, p_n\}$ is a finite set of places, $T = \{t_1, t_2, t_3, \ldots, t_m\}$ is a finite set of transitions. $P_r(p, t)$ is a mapping $P \times T \rightarrow \{0, 1\}$ marked as an arrow diagram with directions from places to transitions. $P_o(t, p)$ is a mapping marked as an arrow diagram with directions from transitions to places.

The marking m is a function $m : P \rightarrow \mathbb{N}$ identified with a multiset on P which can be seen also as a vector $\mathbb{N}^{n_p}$. A transition t is enabled in marking m if and only if $m(p) \geq I(p, t)$, $\forall p \in P$ where $m(p)$ represents the number of tokens in place p in marking m. Enabled transitions may fire, so that the firing of transition t in marking m yields a new marking $m' = m + c \cdot \sigma$. The firing of a sequence σ of transitions enabled at m and yielding m' is denoted as $m[\sigma\rangle m'$

A Stochastic Petri Net (SPN) is a quadruple SPN $= (\mathrm{PN}, m_0, \lambda, \Pi)$ where $m_0 : P \rightarrow \mathbb{N}$ is a multiset on P representing the initial marking, $\lambda = (\lambda_0, \lambda_1, \ldots, \lambda_n)$ is a numerous firing rates connected with transitions, and $\Pi : T \rightarrow \mathbb{N}$ is a priority function that maps every transition to a natural number.

An ordinary continuous-time stochastic Petri Net is a Petri Net with a set of positive, finite, and exponentially distributed firing rates, $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, associated with all its transitions. An enabled transition can fire after an exponentially distributed time delay with parameter $1/\lambda$ elapses. Live and bounded Stochastic Petri Nets are isomorphic to continuous-time Markov Chains due to the memoryless property of exponential distribution [12]. This property makes it possible to analyse stochastic Petri Nets and determine some significant performance measures. The markings on the reachability graph represent the state of the Markov Chain, while the exponential firing rates of the transitions in the stochastic Petri Net represent the state transition rates. Performance measures are computed by solving a system of linear equations that represent the Markov Chain. When a transition is activated at marking m in a stochastic Petri Net, the tokens flow based on the input and output functions. The Markov process is obtained by giving the rate of the corresponding transition to each arc in the reachability graph $R(m_0)$. The steady-state probability distribution, $\Pi = (\pi_0, \pi_1, \ldots, \pi_q)$, of a Stochastic Petri Net is obtained by solving a linear system.

$$\Pi A = 0, \quad \sum_{i=0}^{q-1} \Pi_i = 1 \tag{3.1}$$

Where, $A = (a_{ij})_{q \times q}$ is the transition rate matrix. For $i = 0, 1, \ldots, q-1$, A's i-th row elements, (i.e., a_{ij}), $j = 0, 1, \ldots, q-1$ are determined as follows:

(1) If $j \neq i$, a_{ij} is the sum of all outgoing arcs from state m_i to m_j.
(2) Since any row elements in A satisfies $\sum_{j=0}^{q-1} a_{ij} = 0$, then $a_{ii} = -\sum_{j \neq i}^{q-1} a_{ij}$, where a_{ij} represents the sum of firing rates of transitions enabled at m_i, i.e., transition rates leaving state m_i.

From the steady-state distribution Π and transition firing rates λ, the required performance indices of the system modelled by Stochastic Petri Net can be obtained.

SPN is a timed transition Petri Net with atomic firing and all transition delays are exponentially distributed. Generalised Stochastic Petri Nets (GSPN) which is basically SPN with two kinds of transition delays, one is immediate firing and other one is exponential time firing. SPN are isomorphic to continuous time Markov Chain (CTMC). SPN's are Petri Net in which each transition is presumed to fire after a certain amount of time (firing time) has passed since it was enabled. SPN assumes that there delay as random variables with a negative exponential distribution.

An SPN model's operations can be viewed in two ways. When a new marking is inserted, the first means that each enabled transition samples an instance of the random firing delay from the associated Probability distribution function (Pdf). The structural properties of the reachability graph and labelling arcs obtained with the firing rate of the transition whose firing causes the marking transformation are identical, and the identity of the reachability sets implies that the structural properties of the reachability graph and labelling arcs obtained with the firing rate of the transition whose firing causes the marking transformation are identical. At the end of the firing time, tokens are withdrawn from its input locations and deposited in its output places. The number of tokens in the flow is determined by the input and output functions. The Markov process is created by assigning the rate of the corresponding transition to each arc after building the reachability graph $R(m_0)$.

4. CTMC (Continuous time Markov Chain) [Bause]

A CTMC is a discrete state Markov process with a state that can change at any time.

The stochastic process $\{X(t)\}$ forms a continuous-time Markov Chain if for all integers n and for any sequence $t_0, t_1, t_2, \ldots, t_n, t_{n+1}$ with $t_0 < t_1 < \cdots < t_n <$

t_{n+1} we have

$$P\{X(t_{n+1}) = x_{n+1},\, x(t_0) = x_0, x(t_1) = x_1, \ldots, x(t_n) = x_n\}$$
$$= P\{X(t_{n+1}) = x_{n+1} \mid x(t_n) = x_n\}$$

A homogeneous CTMC is represented by a collection of states and an infinitesimal generator matrix A_i where $A_{ij}, i \neq j$ is the exponentially distributed transition rate between states x_i, x_j $-A_{ij}$ is the parameter of the exponential distribution of the sojourn time in state x_i, where

$$A_{ij} = -\sum_{j \neq i}^{m-1} A_{ij}, \quad \text{where } (a_{ij})_{m \times m}. \tag{4.1}$$

The rate of exponential distribution associated with the state-to-state transition is represented by the off-diagonal elements of the transition matrix A, which is a square matrix of order m. The principal diagonal elements are selected in such a way that the elements of each row sum to zero.

Theorem 4.1. [19] *In a finite homogeneous, irreducible CTMC, the limiting probabilities $\{\pi_i\}$ always exist and are independent of the initial probability distribution. Moreover, $\{\pi_i\}$ is also a stationary probability distribution which can be calculated from solving the set of equations $\Pi A = 0$; $\sum_{i=0}^{n} \pi_i = 1$. The necessary performance indices of the system described by the SPN can be determined from the steady-state distribution π and transition firing rates λ.*

5. SPN with Fuzzy Parameter

The exponential representation of activity dimensions, such as signals, messages, and transmitters, is achieved in the conventional Markov technique and SPNs by the use of λ, which is determined statistically from clean data with a confidence level based on firing and is recognized as a constant value. In this communication system, fuzzy steady-state probabilities are obtained by applying fuzzified parameters to stochastic PNs, which are based on the fuzzification of transition firing rates. We present our approach, with the important concepts about fuzzy sets [18],[21],[16].

6. Fuzzy α-Cut

A fuzzy set $\tilde{A}$ on a universal set X can be defined by its membership function

$$\mu_{\tilde{A}} : X \to [0, 1] \tag{6.1}$$

where $\mu_{\tilde{A}}(x)$ defines the membership of element x in fuzzy set $\tilde{A}$.

A real value $\mu_{\tilde{A}}(x)$ in $[0, 1]$ is assigned to each element $x \in X$, we represent for example, the fuzzy set for a triangular fuzzy number denoted by $\tilde{A} = \langle a, m, b \rangle$, where $a \leq m \leq b$,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{m-a} & \text{if } a \leq x \leq m \\ \frac{b-x}{b-m} & \text{if } m \leq x \leq b \\ 0 & \text{if } x \geq b \end{cases}$$

The α-cut of a fuzzy set $\tilde{A}$, denoted by A^{α}, is the crisp set composed of all elements x of the universe of discourse X for which the membership is greater than or equal to α, i.e.,

$$A^{\alpha} = \{x \in X \mid \tilde{A}(x) \geq \alpha\},$$

where α is a parameter in the range $0 \leq \alpha \leq 1$

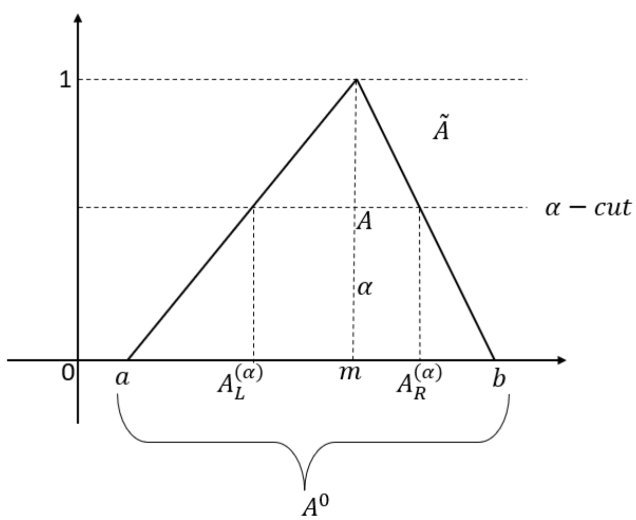


FIGURE 1. α-cut of Fuzzy Number

With the introduction of α-cuts, the confidence interval defined by α-cuts are written as

$$\tilde{A} = [a_{L^{(\alpha)}} \ a_{R^{(\alpha)}}] \quad \text{and} \quad \tilde{B} = [b_{L^{(\alpha)}} \ b_{R^{(\alpha)}}]$$

Then basic fuzzy operations can be done by using the following equations;

$$\tilde{A}^{(\alpha)} + \tilde{B}^{(\alpha)} = [a_{L^{(\alpha)}} + b_{L^{(\alpha)}},\ a_{R^{(\alpha)}} + b_{R^{(\alpha)}}] \tag{6.2}$$
$$\tilde{A}^{(\alpha)} - \tilde{B}^{(\alpha)} = [a_{L^{(\alpha)}} - b_{R^{(\alpha)}},\ a_{R^{(\alpha)}} - b_{L^{(\alpha)}}] \tag{6.3}$$
$$\tilde{A}^{(\alpha)} \cdot \tilde{B}^{(\alpha)} = [C^{(\alpha)},\ d^{(\alpha)}] \tag{6.4}$$
$$C^{(\alpha)} = \min\{a_{L^{(\alpha)}} b_{L^{(\alpha)}},\ a_{L^{(\alpha)}} b_{R^{(\alpha)}},\ a_{R^{(\alpha)}} b_{L^{(\alpha)}},\ a_{R^{(\alpha)}} b_{R^{(\alpha)}}\} \tag{6.5}$$
$$d^{(\alpha)} = \max\{a_{L^{(\alpha)}} b_{L^{(\alpha)}},\ a_{L^{(\alpha)}} b_{R^{(\alpha)}},\ a_{R^{(\alpha)}} b_{L^{(\alpha)}},\ a_{R^{(\alpha)}} b_{R^{(\alpha)}}\} \tag{6.6}$$
$$\tilde{A}^{(\alpha)}/\tilde{B}^{(\alpha)} = [a_{L^{(\alpha)}},\ b_{L^{(\alpha)}}] \cdot [1/b_{R^{(\alpha)}},\ 1/b_{L^{(\alpha)}}] \tag{6.7}$$

When $\tilde{A}^{(\alpha)}/\tilde{B}^{(\alpha)}$, provided that zero does not belong to $\tilde{B}^{(\alpha)}$ for all α. Live and bounded SPNs are isomorphic to CTMC due to the memoryless property of exponential distribution [12].

In our analysis description of exponentially distributed transition firing rates as triangular fuzzy numbers to take in to the consideration of both randomness and fuzziness.

The exponential $E(\lambda)$ has density

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6.8}$$

The mean and variance of $E(\lambda)$ are $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$, respectively. The fuzzy mean (variance) is the fuzzification of the crisp mean (variance). The conditional probability of a fuzzy event 'A' given a fuzzy event B is defined by [19] as

$$\tilde{P}(A \mid B) = \frac{\tilde{P}(A \cdot B)}{\tilde{P}(B)}, \quad \tilde{P}(B) > 0 \tag{6.9}$$

Therefore,Our method is a two-stage modelling approach. The first stage is finding steady state distributions parametrically using equation (3.1) and numeric results of π_i's are calculated. In the second stage, each steady state probability, π_i, is expanded as a function of λ_i. The stochastic nature of the system is crisp. The probability statement of the memoryless property of the crisp exponential is

$$P[X \geq t + \tau \mid X \geq t] = P[X \geq \tau] \tag{6.10}$$

So,

$$P[X \geq t + \tau \mid X > t](\alpha) = \left\{ \frac{\int_T^\infty \lambda e^{-\lambda x}\, dx}{\lambda \in \bar{\lambda}^{(\alpha)}} \right\} \tag{6.11}$$

By fuzzy calculation theory (Zadeh 1998, Buckley 2005), to find the α-cuts of the fuzzy steady state probabilities, we solve an optimization problem which gives the feasible solution.

The procedure to calculate the fuzzy steady state probabilities is as follows:

(1) Model a communication system using a SPN with exponential time delays with transitions.

(2) Generate the reachability graph and label all states of corresponding transition.
(3) Find the steady state probabilities π_i and draw the Markov Chain of the SPN.
(4) Use π_i's obtained in (3) as triangular fuzzy numbers.
(5) Compare the fuzzy steady state probabilities by using equations (6.1) – (6.6) in terms of α-cuts.
(6) For each π_i, the maximum and minimum value ($\alpha = 0$ value) must be in the interval [0,1]. If $\alpha = 0$ each π_i does satisfy this, the result is feasible. If not, then continue the optimization process which is restricted to the interval [0,1].
(7) If the α-cut representation for the fuzzy steady state probability is $\pi_i = [\pi_i^{-(\alpha)}, \pi_i^{+(\alpha)}]$, where $i = 1, 2, \ldots, n$, and n is the number of states. Then the Linear Programming Problem (LPP) is

Minimize $Z = \alpha$
Subject to

$$\pi_i^{+(\alpha)} \leq 1$$
$$\pi_i^{-(\alpha)} \geq 0$$
$$0 \leq \alpha \leq 1$$
$$\pi_i^{-(\alpha)} \leq \pi_i^{+(\alpha)}$$

In the next section, a numerical example of a communication system of a stochastic Petri Net is illustrated with our approach.

Example of a Communication System

For modeling and analysis of a communication system, stochastic variability and α-cut representation are represented using SPNs in conjunction with fuzzy set theory. In this section, a communication system is selected from "Falko Bause and Pieter Kritzinger (2013) SPN – An introduction to the theory," which is illustrated in Fig. 2.

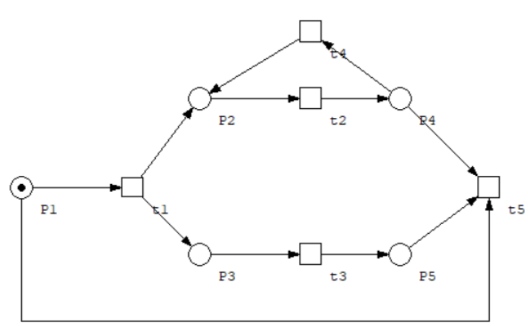


FIGURE 2. PN of Simple Communication System

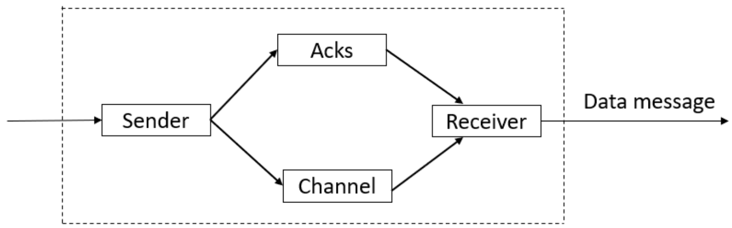


FIGURE 3. Communication Protocol

Consider the Communication Network between the two processes—one shown as the sender and the other as the recipient in Fig. 3. When a message is received, the receiver sends an Ack back to the sender. The sender sends messages to a buffer, which the receiver retrieves. The sender starts processing and transmitting a new message after getting the receiver's Ack. Suppose that the sender takes 2 - time units to send a message to the buffer, 1 unit time to receive the Ack, and 1 unit time to process the next message. The receiver takes 3 - unit time to process a new message and 1 unit time to send back an Ack. The receiver takes again 2 - unit time to get a message from the buffer and a received message, the communication protocol is given in Fig. 3.

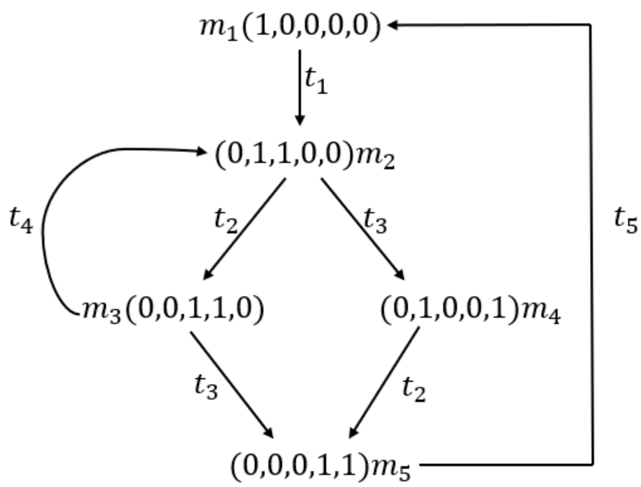


FIGURE 4. Reachability Graph of the SPN of Fig 1.

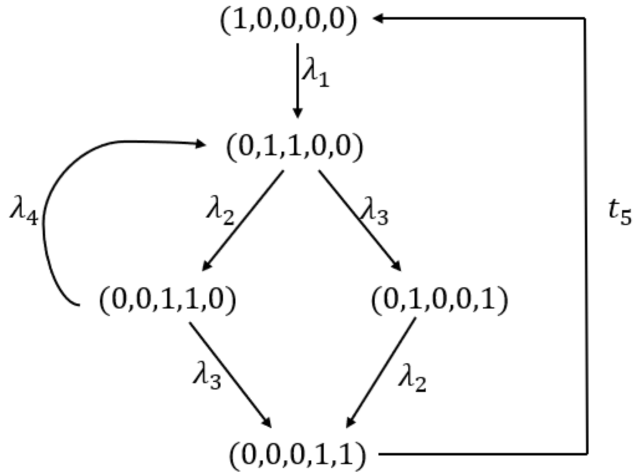


FIGURE 5. Markov chain of the SPN

In the given SPN (Fig. 2), transition t_1 is enabled at $M_0 = (1, 0, 0, 0, 0)$. The time elapsed until t_1 fires is exponentially distributed with rate λ_1 (the average time for t_1 to fire is $\frac{1}{\lambda_1}$). Once t_1 has fired, using the firing rule of PNs, we obtain marking $M_1 = (0, 1, 1, 0, 0)$. At M_1, t_2 and t_3 are concurrently enabled. Based on firing of t_2 and t_3, the markings are given in Fig. 4.

If transition t_2 fires first, the SPN changes to marking $M_2 = (0, 0, 1, 1, 0)$. If t_3 fires before t_2, we set the marking $M_3 = (0, 1, 0, 0, 1)$. The next marking thus depends on which transition fires. The performance or quantitative analysis of SPNs can be carried out straight forwarding by analyzing the corresponding Markovian process. Fig. 4 depicts the reachability graph of the SPN in Fig. 2. The corresponding Markov Chain (MC) with λ_i of transition i as an arc label to each transition in the Markov chain given in Fig. 5. SPN in Fig. 2 displays sequential operation (t_5, t_1), parallel operation (t_2, t_3), forking (t_1), joining (t_5), and conflict (t_4, t_5). Assume mean firing rates $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = 1$, $\lambda_4 = 3$, $\lambda_5 = 2$.

The reachability set has fire markings, or, to say it more simply, there are five states in the Markov chain, beginning with the initial marking of one token in place P_1 and no tokens in the remaining locations. We may determine the steady state probabilities by using the marking probabilities and the total number of tokens in each place in a given marking. Using the infinitesimal generator A of the MC is given by

$$A = \begin{pmatrix} 2 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 3 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Solving the Markov chain using $\pi A = 0$, the steady-state marking probabilities are:

$$P(M_1) = P[(1,0,0,0,0)] = \frac{5}{43}, \quad P(M_2) = P[(0,1,1,0,0)] = \frac{8}{43},$$
$$P(M_3) = P[(0,0,1,1,0)] = \frac{2}{43}, \quad P(M_4) = P[(0,1,0,0,1)] = \frac{22}{43},$$
$$P(M_5) = P[(0,0,0,1,1)] = \frac{5}{43}.$$

By equation (3.1), we obtain the following system of equations:

$$(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) \begin{pmatrix} -\lambda_1 & \lambda_1 & 0 & 0 & 0 \\ 0 & -\lambda_2 - \lambda_3 & \lambda_2 & \lambda_3 & 0 \\ 0 & \lambda_4 & -\lambda_3 - \lambda_4 & 0 & \lambda_3 \\ 0 & 0 & 0 & -\lambda_3 & \lambda_3 \\ \lambda_5 & 0 & 0 & 0 & -\lambda_5 \end{pmatrix} = 0 \qquad (6.12)$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$$

The above system's solution provides the following parametric steady state probabilities in terms of transition firing rates:

$$\mathbf{\Pi}^\top = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \end{pmatrix} = \begin{pmatrix} \frac{5\lambda_2\lambda_3\lambda_4\lambda_5}{\lambda} \\ \frac{8\lambda_1\lambda_3\lambda_4\lambda_5}{\lambda} \\ \frac{2\lambda_1\lambda_2\lambda_4\lambda_5}{\lambda} \\ \frac{8\lambda_1\lambda_2\lambda_3\lambda_5}{\lambda} \\ \frac{5\lambda_1\lambda_2\lambda_3\lambda_4}{\lambda} \end{pmatrix} \qquad (6.13)$$

Where,

$$5\lambda_2\lambda_3\lambda_4\lambda_5 + 8\lambda_1\lambda_3\lambda_4\lambda_5 + 2\lambda_1\lambda_2\lambda_4\lambda_5 + 8\lambda_1\lambda_2\lambda_3\lambda_5 + 5\lambda_1\lambda_2\lambda_3\lambda_4 = 1$$

After obtaining steady state probabilities in terms of transition firing rates, the second step of our approach requires us to express the transition firing rates as triangular fuzzy numbers. The fuzzy number values for each transition firing rate are shown in Table 2.

TABLE 1. Places, Transitions and their firing rates used in the model.

Places	Interpretation	Transition	Firing rates
P_1	Forking	t_1	$\lambda_1 = 2$
P_2	Constantly enabled	t_2	$\lambda_2 = 1$
P_3	Constantly enabled	t_3	$\lambda_3 = 1$
P_4	Conflict	t_4	$\lambda_4 = 3$
P_5	Joining	t_5	$\lambda_5 = 2$

TABLE 2. Transition firing rates and their α-cut representation.

Fuzzy λ Value	α-cut representation
$\lambda_1 = (\|1\|2\|3)$	$\lambda_1 = (1 + \alpha; 3 - \alpha)$
$\lambda_2 = (\|0.9\|1\|1.1)$	$\lambda_2 = (0.9 + 0.1\alpha; 1.1 - 0.1\alpha)$
$\lambda_3 = (\|0.9\|1\|1.1)$	$\lambda_3 = (0.9 + 0.1\alpha; 1.1 - 0.1\alpha)$
$\lambda_4 = (\|2\|3\|4)$	$\lambda_4 = (2 + \alpha; 4 - \alpha)$
$\lambda_5 = (\|1\|2\|3)$	$\lambda_5 = (1 + \alpha; 3 - \alpha)$

By replacing the fuzzy values from Table 2 in the parametric steady-state probability representation equation (6.13) and applying fuzzy mathematics as given in equations (6.2) to (6.7), the following α-cut representations of fuzzy steady-state probabilities are obtained:

$$\pi_1(\alpha) = \left(\frac{0.05\alpha^4 + 1.05\alpha^3 + 6.85\alpha^2 - 13.95\alpha - 8.1}{1.18\alpha^4 - 16.7\alpha^3 + 60.14\alpha^2 + 93.3\alpha + 40.68}, \frac{-0.05\alpha^4 + 0.35\alpha^3 + 5.45\alpha^2 - 42.35\alpha + 72.6}{0.68\alpha^4 - 24.34\alpha^3 + 183.76\alpha^2 - 570.82\alpha - 628.32} \right)$$

$$\pi_2(\alpha) = \left(\frac{0.8\alpha^4 + 1.04\alpha^3 + 28.8\alpha^2 + 41.6\alpha + 14.4}{1.18\alpha^4 - 16.7\alpha^3 + 60.14\alpha^2 + 93.3\alpha + 40.68}, \frac{-0.8\alpha^4 - 16.8\alpha^3 + 114.4\alpha^2 - 319.2\alpha + 316.8}{0.68\alpha^4 - 24.34\alpha^3 + 183.76\alpha^2 - 570.82\alpha - 628.32} \right)$$

$$\pi_3(\alpha) = \left(\frac{0.2\alpha^4 + 2.6\alpha^3 + 8.2\alpha^2 + 9.4\alpha + 3.6}{1.18\alpha^4 - 16.7\alpha^3 + 60.14\alpha^2 + 93.3\alpha + 40.68}, \frac{-0.2\alpha^4 - 4.2\alpha^3 + 28.6\alpha^2 - 79.8\alpha + 79.2}{0.68\alpha^4 - 24.34\alpha^3 + 183.76\alpha^2 - 570.82\alpha - 628.32} \right)$$

$$\pi_4(\alpha) = \left(\frac{0.08\alpha^4 + 1.6\alpha^3 + 9.44\alpha^2 + 14.4\alpha + 6.48}{1.18\alpha^4 - 16.7\alpha^3 + 60.14\alpha^2 + 93.3\alpha + 40.68}, \frac{0.08\alpha^4 - 2.24\alpha^3 + 20.96\alpha^2 - 73.92\alpha + 87.12}{0.68\alpha^4 - 24.34\alpha^3 + 183.76\alpha^2 - 570.82\alpha - 628.32} \right)$$

$$\pi_5(\alpha) = \left(\frac{0.05\alpha^4 + 1.05\alpha^3 + 6.85\alpha^2 + 13.95\alpha + 8.1}{1.18\alpha^4 - 16.7\alpha^3 + 60.14\alpha^2 + 93.3\alpha + 40.68}, \frac{0.08\alpha^4 - 2.24\alpha^3 + 20.96\alpha^2 - 73.92\alpha + 87.12}{0.05\alpha^4 - 1.45\alpha^3 + 14.35\alpha^2 - 55.55\alpha + 72.6} \right)$$

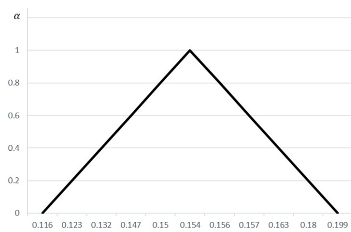
The graph of the fuzzy steady-state probabilities are given in Fig.6(a) - Fig.6(e).

Although for each α, the maximum and minimum values ($\alpha = 0$ value) must be in the interval [0,1], it can be seen that π_2^+ and π_4^+ do not satisfy this condition. So, we must optimize it to find the minimum α-cut that satisfies the condition. Since $\pi_1 = \pi_5$ and $\pi_1^-\alpha\& \geq 0$the optimization problem can be reduced to the following:

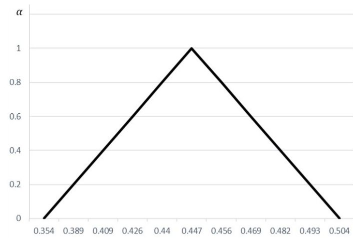
Minimize $Z = \alpha$

Subject to

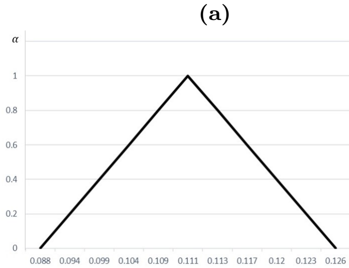
$$\pi_1^{+(\alpha)} \leq 1$$
$$\pi_2^{+(\alpha)} \leq 1$$
$$\pi_3^{+(\alpha)} \leq 1$$
$$\pi_3^{+(\alpha)} \leq 1$$
$$\pi_1^{+(\alpha)}, \pi_2^{+(\alpha)}, \pi_3^{+(\alpha)}, \pi_4^{+(\alpha)} \geq 0$$



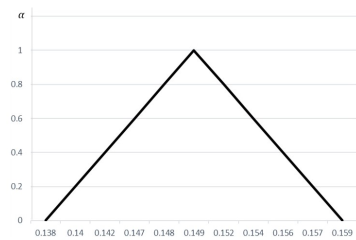
(a)



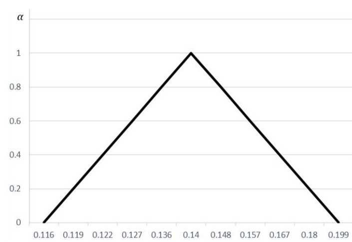
(b)



(c)



(d)



(e)

Figure 6: The Graphical representation of fuzzy - steady state probabilities (a)π_1, (b)π_2, (c)π_3, (d)π_4, (e)π_5

The solution for the optimization problem (6.14) is 0.51 obtained by using MATLAB. This 'α' value is the one that makes the fuzzy steady-state probabilities feasible. The largest interval of final fuzzy steady-state probabilities are represented by a cut value of $\alpha = 0$, while the crisp SPN probability is represented by a cut value of $\alpha = 1$ given in Table 3. As a concluding remark regarding uncertainty represented by fuzzy numbers, it should be pointed out that in some cases, fuzzy numbers are asymmetric as they tend to indicate where the time value could be located.

Using the appropriate simplifications, the problem is as follows:

Minimize $Z = \alpha$

Subject to

$$\begin{aligned} 0.73\alpha^4 + 24.69\alpha^3 - 178.31\alpha^2 + 468.47\alpha - 555.72 &\leq 0 \\ 0.12\alpha^4 - 7.54\alpha^3 + 69.36\alpha^2 - 251.62\alpha + 311.52 &\leq 0 \\ 0.88\alpha^4 - 20.14\alpha^3 + 155.1\alpha^2 - 491.02\alpha + 549.12 &\leq 0 \\ 0.64\alpha^4 + 22.06\alpha^3 - 162.74\alpha^2 + 496.9\alpha - 541.2 &\leq 0 \end{aligned} \tag{6.14}$$

Here the marking M_1 at which P_2 and P_3 is marked with state probability π_1, π_2 within α cut as (0.116/0.154/0.19), (0.354/0.447/0.504). The crisp case with a single value does not enable us to have a unique result where the steady state probability using SPN together with fuzzy set theory in system modelling are unavoidable.

7. Conclusion

In this paper, we presented a method for applying stochastic petri nets with fuzzy parameters to model and analyse discrete-event (communication) systems. Our two-stage process, which is based on fuzzy sets and PNs, aims to improve the modelling and analysis capabilities of complex systems. In the description above, it is unreasonably assumed that every message is transmitted error-free. In reality, transmission faults are inevitable, especially when noise is present in any communication channel. On the other hand, symmetric revealed that errors are equally likely to occur regardless of the character being conveyed, the challenge of determining an unknown Markov chain's stationary probability from a series of observations. Fuzzy numbers should be used to estimate transition probabilities and carry out the calculations to get fuzzy stationary probability. It should be noted that fuzzy numbers can occasionally be asymmetrical in their representation of uncertainty because they have a tendency to suggest possible locations for the genuine value. True crisp stationary probabilities can be more accurately approximated with defuzzification. The suggested methodology, which was tested on a communication system, can be used to the modelling and analysis of any complex, dynamic, and time-sensitive system that SPNs are used to simulate. For future research, analysis with respect to fuzzy parameter and application of the proposed approach in other fields rather than communication systems are recommended.

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