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ANALYSIS OF THE STOCHASTIC OSKOLKOV SYSTEM WITH A MULTIPOINT INITIAL-FINAL VALUE CONDITION

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Abstract. Recently, the theory of stochastic equations has been actively developing. Here it is worth noting the classical direction of research by Ito – Stratonovich – Skorokhod.Its main problem is to overcome the difficulties associated with the differentiation of a non-differentiable (in "the usual sense") Wiener process. It is also necessary to note the approach of I.V. Melnikova, within the framework of which stochastic equations are considered in Schwarz spaces using the generalized derivative. Our research will use methods and results of the theory, which is based on the concept of the Nelson – Glicklich derivative. Most studies consider the Cauchy problem for stochastic equations. In this article, instead of the Cauchy condition, it is proposed to consider a multipoint initial-final value condition. The obtained abstract results are used to analyze the solvability of the stochastic Oskolkov system, which models the dynamics of the velocity and pressure of a viscoelastic incompressible fluid. It is considered with a no-slip boundary condition and a multipoint initial-final value condition. The main result of the article is the proof of the solvability of the posed problem.

Introduction

Let $\Omega \subset \mathbb{R}^n$, $n \in \mathbb{N} \setminus \{1\}$, be a bounded domain with boundary $\partial\Omega$ of the class C^∞. In the cylinder $\Omega \times \mathbb{R}$ consider the system of equations

$$(1 - æ\nabla^2)v_t = \nu\nabla^2 v - (v \cdot \nabla)v - \nabla p + f, \quad \nabla \cdot v = 0, \tag{0.1}$$

modeling the dynamics of velocity $v = (v_1, v_2, \ldots, v_n)$, $v_k = v_k(x,t)$, $k = \overline{1,n}$, and pressure $p = p(x,t)$, $(x,t) \in \Omega \times \mathbb{R}$, viscoelastic incompressible fluid. Here the parameter $\nu \in \mathbb{R}_+$ characterizes the viscous properties of the liquid, and the parameter $æ \in \mathbb{R}$ characterizes the elastic properties of the fluid. The prototype of such a liquid is high-paraffin type of oil, produced, in particular, in the fields of Western Siberia. The system (0.1) was first obtained and studied by A.P. Oskolkov [1]. Therefore, it later received the name "*Oskolkov system*" [2]. In [3]

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G.A. Sviridyuk began studying a modified system of equations (0.1)

$$(\lambda - \nabla^2)u_t = \nu\nabla^2 u - \alpha(u\cdot\nabla)u - \nabla p, \quad \nabla\cdot u = 0. \tag{0.2}$$

To solve the system of equations (0.2) consider "the condition no-slip" to the boundary of the domain

$$u(x,t) = 0, \ (x,t) \in \partial\Omega\times\mathbb{R}. \tag{0.3}$$

As is known, Newton rheological relation, which models the dynamics of viscous incompressible fluids, has the form [4], [5], [6]

$$\sigma = 2\nu D - p\mathbb{I}. \tag{0.4}$$

Here σ and D are the stress and strain rate tensors, respectively, $\nu \in \mathbb{R}_+$ is the viscosity coefficient, $\mathbb{I}$ is the unit matrix, p characterizes the pressure. After substituting (0.4) into the equations of motion of a continuous incompressible medium in the Cauchy form

$$v_t = \nabla\cdot\sigma, \quad \nabla\cdot v = 0, \tag{0.5}$$

we obtain the famous Navier – Stokes system of equations

$$v_t = \nu\nabla^2 v - (v\cdot\nabla)v - \nabla p, \quad \nabla\cdot v = 0, \tag{0.6}$$

modeling the evolution of velocity and pressure of a viscous incompressible fluid. Extensive literature is devoted to the study of equations (0.6) in various aspects. Let us note here the fundamental monographs of O.A. Ladyzhenskaya [7] and R. Temam [8].

Various effects (for example, the recoil effect or the fading memory effect) that arise when pumping oil through pipelines and do not fit into the framework of the model (0.6) have prompted many researchers to revise the relationship (0.4). In particular, the rheological relation by V.A. Pavlovskii [9]

$$\sigma = 2\nu D + æ D_t - p\mathbb{I}, \tag{0.7}$$

was offered and experimentally tested [10]. Here the coefficient æ has the physical meaning of relaxation viscosity, and by design it is strictly positive. It immediately follows from (0.7) that the fluid velocity in the absence of stress does not immediately become equal to zero, as in Newton's model (0.4), but tends to zero exponentially, which gives the required relaxation effect.

However, later, in experiments with aqueous solutions of polymers, it turned out that the constant æ can also take negative values [11]. Moreover, for negative values of æ, the model (0.7) demonstrates strong instability. Analysis of the relationship (0.7), carried out in [12] from the standpoint of the rheological theory [4], [5], [6], showed that in the case of $æ \in \mathbb{R}_+$ this model reveals properties characteristic of solids. Consequently, it cannot represent any non-Newtonian fluid [4], and therefore in [12] it is proposed to call such objects *media*.

If the rheological relation (0.7) is substituted into the equation (0.5), we obtain a system of equations (0.1), which, after simple algebraic transformations, will turn into the system (0.2). Here $\alpha = \lambda = æ^{-1}$ are parameters characterizing elastic properties of liquid. In what follows, the system (0.2) will be considered for different values of the parameters α and λ. Initially, various initial-boundary value problems for the equations (0.1) were studied by A.P. Oskolkov [1], [2], [13], [14],

[15], then his students [16], [17] joined the research. The results of their research consisted mainly in proving the unique solvability of various initial-boundary value problems for the system (0.1) and its various generalizations for *positive values* of the parameter æ in bounded and unbounded regions of space $\mathbb{R}^n$, $n \in \{2, 3, 4\}$.

Relevant equations [15] in specially constructed spaces [18] can be reduced to an abstract model

$$L\dot{u} = Mu + N(u),$$

where L, M are linear, and N are nonlinear operators. Let's consider the linear abstract model

$$L\dot{u} = Mu + f,$$

in Banach spaces $\mathfrak{U}$ and $\mathfrak{F}$, and the operators $L \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$ (i.e. linear and continuous), $M \in \mathcal{C}l(\mathfrak{U}; \mathfrak{F})$ (i.e. linear, closed and densely defined). Let the operator M be (L, p)-bounded [19], and its L-spectrum satisfy the condition [20]

$$\left\{\begin{array}{c} \sigma^L(M) = \bigcup\limits_{j=0}^{m} \sigma_j^L(M),\ m \in \mathbb{N},\ \text{and } \sigma_j^L(M) \neq \emptyset,\ \text{there exists} \\ \text{a closed contour } \gamma_j \subset \mathbb{C} \text{ bounding the domain } D_j \supset \sigma_j^L(M) \\ \text{such that } \overline{D_j} \cap \sigma_0^L(M) = \emptyset,\ \overline{D_k} \cap \overline{D_l} = \emptyset\ \forall j, k, l = \overline{1, m}, k \neq l. \end{array}\right. \tag{0.8}$$

Then there are relative spectral projectors

$$P_j = \frac{1}{2\pi i} \int_{\gamma_j} R_\mu^L(M) d\mu \in \mathcal{L}(\mathfrak{U}); \quad Q_j = \frac{1}{2\pi i} \int_{\gamma_j} L_\mu^L(M) d\mu \in \mathcal{L}(\mathfrak{F}), j = \overline{0, m},$$

where the boundary contours γ_r are defined by (0.8). We will discuss these projectors in more detail in the third paragraph.

The work is devoted to the study of the stochastic linear Sobolev type equation

$$L \overset{\circ}{\eta} = M\eta + N\omega, \tag{0.9}$$

where $\eta = \eta(t)$ is the required one, and $\omega = \omega(t)$ is a given stochastic $\mathbf{K}$-process ($\mathbf{K}$-"noise"), with multipoint initial-final value condition

$$\lim_{t \to 0+} P_0(\eta(t) - \xi_0) = 0, \quad P_j(\eta(\tau_j) - \xi_j) = 0, \quad j = \overline{1, m}. \tag{0.10}$$

A detailed description will be given in the second paragraph.

The article, in addition to the introduction and bibliography, contains three parts. In the first part, spaces of differentiable random processes with values in a separable Hilbert space are constructed. Moreover, by derivative we mean the Nelson – Gliklich derivative [21], [22], [23], [24]. We call random processes that have Nelson – Glicklich derivatives differentiable "noises" [25], [26], [27], [28]. The second part of the article presents results on the solvability of the stochastic problem (0.9), (0.10) under the condition that the operator M, $p \in \{0\} \cup \mathbb{N}$, is (L, p)-bounded, and a condition guaranteeing the existence of relative spectral projectors P_j, $j = \overline{0, n}$, [29]. These results generalize and develop the abstract results of the works [25], [26], [27], [28]. The third part contains applications of the obtained abstract results for the stochastic Oskolkov system. The list of references does not pretend to be complete and reflects only the tastes and preferences of the authors.

1. Spaces of differentiable "noises"

Let $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ is a complete probability space with a probability measure $\mathbf{P}$ associated with the σ-algebra $\mathcal{A}$ of subsets of the set Ω, and $\mathbb{R}$ is a set of real numbers, endowed with a Borel σ-algebra. A measurable mapping $\xi : \Omega \to \mathbb{R}$ is called *random variable.* A set of random variables whose mathematical expectation is zero, and the variance is finite, forms the Hilbert space $\mathbf{L_2} = \{\xi : \mathbf{E}\xi = 0,\ \mathbf{D}\xi < +\infty\}$ with the scalar product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$ and the norm $\|\xi\|^2_{\mathbf{L_2}} = \mathbf{D}\xi$. Note that in $\mathbf{L_2}$ the orthogonality of the vectors ξ and η (i.e. $(\xi, \eta) = 0$) is equivalent to correlated random variables ξ and η. Indeed, $0 = \mathrm{cov}(\xi, \eta) = \mathbf{E}\xi\eta = (\xi, \eta) = 0$.

Let us take the set $\mathfrak{I} \subset \mathbb{R}$ and consider two mappings: $f : \mathfrak{I} \to \mathbf{L_2}$, which each $t \in \mathfrak{I}$ assigns a random variable $\xi \in \mathbf{L_2}$, and $g : \mathbf{L_2} \times \Omega \to \mathbb{R}$, which assigns to each pair (ξ, ω) point $\xi(\omega) \in \mathbb{R}$. Display $\eta : \mathfrak{I} \times \Omega \to \mathbb{R}$, which has the form $\eta = \eta(t, \omega) = g(f(t), \omega)$, we call *(one-dimensional) stochastic process.* For every fixed $t \in \mathfrak{I}$ value of the stochastic process $\eta = \eta(t, \cdot)$ is a random variable, i.e. $\eta(t, \cdot) \in \mathbf{L_2}$, which we call *cross section* of the stochastic process at point $t \in \mathfrak{I}$. For each fixed $\omega \in \Omega$ the function $\eta = \eta(\cdot, \omega)$ is called *(selective) trajectory* of a random process corresponding to the elementary outcome $\omega \in \Omega$. Trajectories are also called *realizations* or *sample functions* of a random process. Usually, when this does not lead to ambiguity, the dependence of $\eta(t, \omega)$ on ω is not indicated and the random process is simply denoted by $\eta(t)$.

Considering $\mathfrak{I} \subset \mathbb{R}$ to be an interval, we call the stochastic process $\eta = \eta(t)$, $t \in \mathfrak{I}$, *continuous,* if a.s. (almost surely) all its trajectories are continuous (i.e. for almost all $\omega \in \mathcal{A}$ trajectories $\eta(\cdot, \omega)$ are continuous functions). A set of continuous stochastic processes forms Banach space, which we denote by the symbol $\mathbf{C}(\mathfrak{I}; \mathbf{L_2})$ with the norm $\|\eta\|_{\mathbf{CL_2}} = \sup\limits_{t \in \mathfrak{I}} (\mathbf{D}\eta(t, \omega))^{1/2}$. Let $\mathcal{A}_0$ be a σ-subalgebra σ-algebras $\mathcal{A}$. Let us construct the subspace $\mathbf{L^0_2} \subset \mathbf{L_2}$ random variables measurable with respect to $\mathcal{A}_0$. Let us denote by $\Pi : \mathbf{L_2} \to \mathbf{L^0_2}$ – ortho projector. Let $\xi \in \mathbf{L_2}$, then $\Pi\xi$ is called *conditional mathematical expectation* of the random variable ξ and is denoted by the symbol $\mathbf{E}(\xi|\mathcal{A}_0)$. Let us fix $\eta \in \mathbf{C}(\mathfrak{I}; \mathbf{L_2})$ and $t \in \mathfrak{I}$, by $\mathcal{N}^\eta_t$ we denote the *σ-algebra generated by random variable* $\eta(t)$, and denote $\mathbf{E}^\eta_t = \mathbf{E}(\cdot|\mathcal{N}^\eta_t)$.

Example 1.1. Wiener process describing Brownian motion in the Einstein – Smoluchowski model (see [22])

$$\beta(t, \omega) = \sum_{k=0}^{\infty} \xi_k(\omega) \sin \frac{\pi}{2}(2k+1)t,\ \ t \in \{0\} \cup \mathbb{R}_+,$$

is a continuous stochastic process. Here the coefficients $\{\xi_k = \xi_k(\omega)\} \subset \mathbf{L}_2$ are pairwise uncorrelated Gaussian random variables such that $\mathbf{D}\xi^2_k = \left[\frac{\pi}{2}(2k+1)\right]^{-2}$, $k \in \{0\} \cup \mathbb{N}$.

Definition 1.2. [21], [22] Let $\eta \in \mathbf{C}(\mathfrak{I}; \mathbf{L_2})$. By the *Nelson – Glicklich derivative* $\overset{\circ}{\eta}$ *stochastic process* η *at point* $t \in \mathfrak{I}$ a random variable

$$\overset{\circ}{\eta}(t, \cdot) = \frac{1}{2} \lim_{\Delta t \to 0+} \mathbf{E}^\eta_t \left(\frac{\eta(t + \Delta t, \cdot) - \eta(t, \cdot)}{\Delta t} \right) + \frac{1}{2} \lim_{\Delta t \to 0+} \mathbf{E}^\eta_t \left(\frac{\eta(t, \cdot) - \eta(t - \Delta t, \cdot)}{\Delta t} \right),$$

is called, if the limit exists in the sense of a uniform metric on $\mathbb{R}$.

If the Nelson – Glicklich derivatives $\overset{\circ}{\eta}(t,\cdot)$ of the stochastic process $\eta(t,\cdot)$ exist in all (or almost all) points of the interval $\mathfrak{I}$, then we talk about the existence of the Nelson – Glicklich derivative $\overset{\circ}{\eta}(t,\cdot)$ on $\mathfrak{I}$ (a.s. on $\mathfrak{I}$.)

Set of continuous stochastic processes having continuous Nelson – Glicklich derivatives $\overset{\circ}{\eta}$ forms a Banach $\mathbf{C}^1(\mathfrak{I};\mathbf{L_2})$ space with the norm

$$\|\eta\|_{\mathbf{C}^1\mathbf{L}_2} = \sup_{t\in\mathfrak{I}}\Big(\mathbf{D}\eta(t,\omega) + \mathbf{D}\overset{\circ}{\eta}(t,\omega)\Big)^{1/2}.$$

We further define by induction the Banach spaces $\mathbf{C}^l(\mathfrak{I};\mathbf{L_2})$, $l\in\mathbb{N}$, stochastic processes whose trajectories a.s. differentiable with respect to Nelson – Gliklich on $\mathfrak{I}$ up to order $l\in\{0\}\cup\mathbb{N}$ inclusive [30]. The norms in them are given by the formulas $\|\eta\|_{\mathbf{C}^l\mathbf{L}_2} = \sup\limits_{t\in\mathfrak{I}}\left(\sum\limits_{k=0}^{l}\mathbf{D}\overset{\circ}{\eta}{}^{(k)}(t,\omega)\right)^{1/2}$. Here we will consider the zero-order Nelson – Gliklich derivative to be the original random process, i.e. $\overset{\circ}{\eta}{}^{(0)}\equiv\eta$, and under the Nelson – Gliklich derivative are of order k we will understand the Nelson – Gliklich derivative of the first order from the Nelson – Gliklich derivative of order $k-1$. For brevity we will call *spaces of differentiable "noises"* (see [25], [26], [27], [28]).

Example 1.3. In [22, 30] it is shown that $\beta\in\mathbf{C}^l(\mathbb{R}_+;\mathbf{L_2})$, $l\in\{0\}\cup\mathbb{N}$, and $\overset{\circ}{\beta}(t) = \dfrac{\beta(t)}{2t}$, $t\in\mathbb{R}_+$.

Thus, spaces of random variables $\mathbf{L}_2$ and spaces of differentiable "noises" $\mathbf{C}^l(\mathfrak{I};\mathbf{L}_2)$, $l\in\{0\}\cup\mathbb{N}$. Let's move on to constructing a space *of random* $\mathbf{K}$-*variables.* Take $\mathfrak{H}$ is a separable Hilbert space with an orthonormal basis $\{\varphi_k\}$, a monotone sequence $\mathbf{K}=\{\lambda_k\}\subset\mathbb{R}_+$ such that that $\sum\limits_{k=1}^{\infty}\lambda_k^2<+\infty$, as well as a sequence $\{\xi_k\}=\xi_k(\omega)\subset\mathbf{L}_2$ of random variables such that that $\|\xi_k\|_{\mathbf{L}_2}\le C$, for all $C\in\mathbb{R}_+$, for all $k\in\mathbb{N}$.

Let us construct a $\mathfrak{H}$-*valued random* $\mathbf{K}$-*variable* $\xi(\omega)=\sum\limits_{k=1}^{\infty}\lambda_k\xi_k(\omega)\varphi_k$. Completion of the linear hull of the set $\{\lambda_k\xi_k\varphi_k\}$ by the norm

$$\|\eta\|^2_{\mathbf{H_K}\mathbf{L}_2} = \left(\sum_{k=1}^{\infty}\lambda_k^2\mathbf{D}\xi_k\right)^{1/2}$$

is called the *space of ($\mathfrak{H}$-valued) random* $\mathbf{K}$-*variables* and is denoted by the symbol $\mathbf{H_K}\mathbf{L}_2$. How easy it is to see the space $\mathbf{H_K}\mathbf{L}_2$ is Hilbertian, and the random $\mathbf{K}$-variable constructed above $\xi=\xi(\omega)\in\mathbf{H_K}\mathbf{L}_2$. Likewise, Banach space ($\mathfrak{H}$-*valued*) $\mathbf{K}$- *"noises"* $\mathbf{C}^l(\mathfrak{I};\mathbf{H_K}\mathbf{L}_2)$, $l\in\{0\}\cup\mathbb{N}$, we define as the completion of the linear hull of the set $\{\lambda_k\eta_k\varphi_k\}$ by the norm

$$\|\eta\|^2_{\mathbf{C}^l\mathbf{H_K L_2}} = \sup_{t\in\Im}\left(\sum_{k=1}^{\infty}\lambda_k^2\sum_{m=1}^{l}\mathbf{D}\overset{\circ}{\eta}{}_k^m\right)^{1/2},$$

where the sequence of "noises" $\{\eta_k\} \subset \mathbf{C}^l(\Im;\mathbf{L}_2)$, $l \in \{0\}\cup\mathbb{N}$. As is easy to see, the vector $\eta(t,\omega) = \sum\limits_{k=1}^{\infty}\lambda_k\eta_k(t,\omega)\varphi_k$ lies in the space $\mathbf{C}^l(\Im;\mathbf{H_K L_2})$, if a sequence of vectors $\{\eta_k\} \subset \mathbf{C}^l(\Im;\mathbf{L}_2)$ and all their Nelson – Glicklich derivatives up to order $l \in \{0\}\cup\mathbb{N}$ inclusive are uniformly bounded by the norm $\|\cdot\|_{\mathbf{C}^l\mathbf{L}_2}$.

Example 1.4. Vector lying in all spaces $\mathbf{C}^l(\mathbb{R}_+;\mathbf{H_K L_2})$, $l \in \{0\}\cup\mathbb{N}$,

$$W_{\mathbf{K}}(t,\omega) = \sum_{k=1}^{\infty}\lambda_k\beta_k(t,\omega)\varphi_k,$$

where $\{\beta_k\} \subset \mathbf{C}^l(\Im;\mathbf{L}_2)$ is sequence of Brownian motions, called *($\mathfrak{H}$-valued) Wiener* $\mathbf{K}$*-process.*

2. The multipoint initial-final value condition

Now let $\mathfrak{U}$ ($\mathfrak{F}$) be a real separable Hilbert space with an orthonormal basis $\{\varphi_k\}$ ($\{\psi_k\}$). Let us introduce into consideration a monotone sequence $\mathbf{K} = \{\lambda_k\} \subset \{0\}\cup\mathbb{R}$ such that $\sum\limits_{k=1}^{\infty}\lambda_k^2 < +\infty$. The symbol $\mathbf{U_K L_2}$ ($\mathbf{F_K L_2}$) denotes the Hilbert space, which is the completion of the linear hull of *random* $\mathbf{K}$*-variables*

$$\xi = \sum_{k=1}^{\infty}\lambda_k\xi_k\varphi_k,\ \xi_k \in \mathbf{L_2},\quad \left(\zeta = \sum_{k=1}^{\infty}\mu_k\zeta_k\psi_k,\ \zeta_k \in \mathbf{L_2}\right),$$

according to the norm $\|\eta\|^2_{\mathbf{U}} = \sum\limits_{k=1}^{\infty}\lambda_k^2\mathbf{D}\xi_k \quad \left(\|\omega\ |^2_{\mathbf{F}} = \sum\limits_{k=1}^{\infty}\mu_k^2\mathbf{D}\zeta_k\right)$. Note that in different spaces ($\mathbf{U_K L}_2$ and $\mathbf{F_K L}_2$) the sequence $\mathbf{K}$ can be different ($\mathbf{K} = \{\lambda_k\}$ in $\mathbf{U_K L}_2$ and $\mathbf{K} = \{\mu_k\}$ in $\mathbf{F_K L}_2$), however, all sequences marked with $\mathbf{K}$ must be monotonic and summable with square. All results, generally speaking, will be true for different sequences $\{\lambda_k\}$ and $\{\mu_k\}$, but for the sake of simplicity we will limit ourselves to the case $\lambda_k = \mu_k$.

Lemma 2.1. *Operator $A \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ exactly when $A \in \mathcal{L}(\mathbf{U_K L}_2;\mathbf{F_K L}_2)$.*

How easy it is to see

$$\|A\xi\|_{\mathbf{F}} \le \sum_{k=1}^{\infty}\lambda_k^2\mathbf{D}\xi_k\|A\varphi_k\|^2_{\mathfrak{F}} \le \text{const}\sum_{k=1}^{\infty}\lambda_k^2\mathbf{D}\xi_k = \text{const}\|\xi\|_{\mathbf{U}}.$$

Lemma 2.2. *Operator $M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ is σ-bounded with respect to operator $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ exactly when $M \in \mathcal{L}(\mathbf{U_K L}_2;\mathbf{F_K L}_2)$ is σ-bounded with respect to the operator $L \in \mathcal{L}(\mathbf{U_K L}_2;\mathbf{F_K L}_2)$. Moreover, the L-spectrum of the operator M coincide in both cases.*

The proof of Lemma 2.2 is similar to the proof of Lemma 2.1 and is therefore omitted. Ideas and methods of theory regarding σ-bounded operators can be found, for example, in [19]. According to this theory, in the case of a (L,σ)-bounded operator M there is a pair of relative spectral projectors

$$P = \frac{1}{2\pi i}\int_{\gamma} R^L_{\mu}(M)d\mu \in \mathcal{L}(\mathfrak{U}),\ Q = \frac{1}{2\pi i}\int_{\gamma} L^L_{\mu}(M)d\mu \in \mathcal{L}(\mathfrak{F}), \tag{2.1}$$

where $\gamma \subset \mathbb{C}$ is the contour bounding the region containing the L-spectrum $\sigma^L(M)$ of the operator M, and $R^L_{\mu}(M) = (\mu L - M)^{-1}L$ $(L^L_{\mu}(M) = L(\mu L - M)^{-1}$ – right (left) L-resolvent operator M. By Lemma 2.1, we transfer the projectors P and Q to the spaces $\mathbf{U_K L_2}$ and $\mathbf{F_K L_2}$ respectively and introduce into consideration the subspaces $\mathbf{U^0_K L_2} = \ker P$, $\mathbf{F^0_K L_2} = \ker Q$ and $\mathbf{U^1_K L_2} = \operatorname{im} P$, $\mathbf{F^1_K L_2} = \operatorname{im} Q$. We denote by L_k (M_k) the restriction of the operator L (M) to $\mathbf{U^k_K L_2}$ $(\mathbf{F^k_K L_2})$, $k = 0$, 1. Fair

Theorem 2.3. *(splitting theorem).* [28] *Let L, $M \in \mathcal{L}(\mathbf{U_K L_2}; \mathbf{F_K L_2})$, operator M (L,σ)-bounded. Then*

(i) operators L_0, $M_0 \in \mathcal{L}(\mathbf{U^0_K L_2}; \mathbf{F^0_K L_2})$; L_1, $M_1 \in \mathcal{L}(\mathbf{U^1_K L_2}; \mathbf{F^1_K L_2})$;

(ii) operators $M_0^{-1} \in \mathcal{L}(\mathbf{F^0_K L_2}; \mathbf{U^0_K L_2})$, $L_1^{-1} \in \mathcal{L}(\mathbf{F^1_K L_2}; \mathbf{U^1_K L_2})$.

Let us construct the operator $H = L_0^{-1}M_0 \in \mathcal{L}(\mathfrak{U}^0)$ $(G = M_0 L_0^{-1} \in \mathcal{L}(\mathfrak{F}^0))$. We call a (L,σ)-bounded operator M *(L,p)-bounded* if there exists a number $p \in \{0\} \cup \mathbb{N}$ such, that $H^p \neq \mathbb{O}$, and $H^{p+1} = \mathbb{O}$. Note that the number $p \in \{0\} \cup \mathbb{N}$ does not change if the operator H is replaced by the operator G. Now let us take the (L,p)-bounded operator M, $p \in \{0\} \cup \mathbb{N}$, and construct families of operators

$$U^t = \frac{1}{2\pi i}\int_{\gamma} R^L_{\mu}(M)e^{\mu t}d\mu,\ F^t = \frac{1}{2\pi i}\int_{\gamma} L^L_{\mu}(M)e^{\mu t}d\mu, \tag{2.2}$$

where the contour $\gamma \subset \mathbb{C}$ is the same as in (2.1). On $\mathbb{R}$ the families (2.2) are commutative groups whose units are the projectors (2.1). Moreover, the families (2.2) extend to the entire complex plane while preserving the group property.

Let $\tau_0 = 0$, $\tau_j \in \mathbb{R}_+$, $(\tau_{j-1} < \tau_j)$, $j = \overline{1,m}$. Let condition (0.8) be satisfied.

Theorem 2.4. [31] *Let the operator M be (L,p)-bounded, $p \in \{0\} \cup \mathbb{N}$, and the condition* (0.8) *holds. Then there exist families of operators*

$$U^t_j = \frac{1}{2\pi i}\int\limits_{\gamma_j} R^L_{\mu}(M)e^{\mu t}d\mu,\ F^t_j = \frac{1}{2\pi i}\int\limits_{\gamma_j} L^L_{\mu}(M)e^{\mu t}d\mu,\ j = \overline{1,m},$$

$$U^t_0 = U^t - \sum_{k=1}^{m} U^t_k,\ F^t_0 = F^t - \sum_{k=1}^{m} F^t_k,\ t \in \mathbb{R}.$$

Moreover

(i) $U^t U^s_j = U^s_j U^t = U^{s+t}_j$, $F^t F^s_j = F^s_j F^t = F^{s+t}_j$ for all s, $t \in \mathbb{R}$, $j = \overline{0,m}$;

(ii) $U^t_k U^s_l = U^s_l U^t_k = \mathbb{O}$, $F^t_k F^s_l = F^s_l F^t_k = \mathbb{O}$ for all s, $t \in \mathbb{R}$, k, $l = \overline{0,m}$, $k \neq l$.

Remark 2.5. Consider the units $P_j = U^0_j$, $Q_j = F^0_j$, $j = \overline{0,m}$. It can be shown, based on the results of Theorem 2.4, that

(i) $PP_j = P_j P = P_j$, $QQ_j = Q_j Q = Q_j$, $j = \overline{0,m}$;

(ii) $P_k P_l = P_l P_k = \mathbb{O}$, $Q_k Q_l = Q_l Q_k = \mathbb{O}$ k, $l = \overline{0,m}$, $k \neq l$.

By Lemma 2.1, we transfer the projectors P_j, Q_j, $j=\overline{0,m}$, to the spaces $\mathbf{U_K L_2}$ and $\mathbf{F_K L_2}$ respectively and introduce subspaces into consideration $\mathbf{U}^{1j}_{\mathbf{K}}\mathbf{L}_2 = \operatorname{im} P_j$, $\mathbf{F}^{1j}_{\mathbf{K}}\mathbf{L}_2 = \operatorname{im} Q_j$, $j=\overline{0,m}$. By construction $\mathbf{U}^{1}_{\mathbf{K}}\mathbf{L}_2 = \bigoplus\limits_{j=0}^{m}\mathbf{U}^{1j}_{\mathbf{K}}\mathbf{L}_2$, $\mathbf{F}^{1}_{\mathbf{K}}\mathbf{L}_2 = \bigoplus\limits_{j=0}^{m}\mathbf{F}^{1j}_{\mathbf{K}}\mathbf{L}_2$. We denote by L_{1j} (M_{1j}) the restriction of the operator L (M) to $\mathbf{U}^{1j}_{\mathbf{K}}\mathbf{L}_2$ $(\mathbf{F}^{1j}_{\mathbf{K}}\mathbf{L}_2)$, $j=\overline{0,m}$.

Theorem 2.6. (Generalized splitting theorem). [28] *Let $L,\ M \in \mathcal{L}(\mathbf{U_K L_2};\mathbf{F_K L_2})$, operator M is (L,p)-bounded, $p\in\{0\}\cup\mathbb{N}$,*
(i) operators $L_{1j}\in\mathcal{L}(\mathbf{U}^{1j}_{\mathbf{K}}\mathbf{L}_2;\mathbf{F}^{1j}_{\mathbf{K}}\mathbf{L}_2)$, $M_{1j}\in\mathcal{L}(\mathbf{U}^{1j}_{\mathbf{K}}\mathbf{L}_2;\mathbf{F}^{1j}_{\mathbf{K}}\mathbf{L}_2)$, $j=\overline{0,m}$;
(ii) there are operators $L^{-1}_{1j}\in\mathcal{L}(\mathbf{F}^{1j}_{\mathbf{K}}\mathbf{L}_2;\mathbf{U}^{1j}_{\mathbf{K}}\mathbf{L}_2)$, $j=\overline{0,m}$.

Let the operators $L,\ M,\ N \in \mathcal{L}(\mathbf{U_K L_2};\mathbf{F_K L_2})$, Let us consider a linear stochastic equation of Sobolev type (0.9). Let us supply the equation (0.9) multipoint initial-final value condition (0.10)

Let us call the stochastic $\mathbf{K}$-process $\eta\in\mathbf{C}^1(\mathbb{R}_+;\mathbf{L}_2)$ (*classical*) *solution of the equation* (0.9), if a.s. all of it trajectories satisfy the equation (0.9) with some $\mathbf{K}$-"noise" $\omega\in\mathbf{C}(\mathbb{R}_+;\mathbf{L}_2)$ and all $t\in\mathbb{R}_+$. The solution $\eta=\eta(t)$ of the equation (0.9) will be called a *solution of the problem* (0.9), (0.10) if the condition (0.10) for some random $\mathbf{K}$-variables $\xi_k\in\mathbf{U_K L_2}$, $k=\overline{0,l}$.

Theorem 2.7. [28] *Let the operator M is (L,p)-bounded, $p\in\{0\}\cup\mathbb{N}$, and condition* (0.8) *is satisfied. Then for any $\tau_j\in\mathbb{R}_+$, $j=\overline{1,m}$, operator $N\in\mathcal{L}(\mathfrak{U};\mathfrak{F})$, monotonic sequence $K\subset\{\lambda_k\}$ such that $\sum\limits_{k=1}^{\infty}\lambda_k^2<+\infty$, $\mathbf{K}$- "noise" $\omega=\omega(t)$ such that $(\mathbb{I}-Q)Nw\in\mathbf{C}^{p+1}(\mathbb{R}_+;\mathbf{U_K L_2})$ and $QNw\in\mathbf{C}(\mathbb{R}_+;\mathbf{U_K L_2})$, and random $\mathbf{K}$-variables $\xi_j\in\mathbf{U_K L_2}$, $j=\overline{0,m}$, independent of ω, there is a unique solution $\eta\in\mathbf{C}^1(\mathbb{R}_+;\mathbf{U_K L_2})$, problem* (0.9), (0.10), *having the form*

$$\eta(t) = -\sum_{q=0}^{p} H^q M_0^{-1}(\mathbb{I}-Q)\overset{\circ}{\omega}{}^{(q)}(t)+$$
$$+\sum_{j=0}^{m}\left[U_j^{t-\tau_j}\xi_j + \int_{\tau_j}^{t} U_j^{s-\tau_j} L_{1j}^{-1} Q_j N\omega(s)ds\right],\ t\in\mathfrak{I}.$$

Corollary 2.8. *Let all the conditions of Theorem 2.7 be satisfied and $\omega(t) = \overset{\circ}{W}_{\boldsymbol{K}}(t)$. Then for any random $\mathbf{K}$-variables $\xi_j\in\mathbf{U_K L_2}$, $j=\overline{0,m}$, there is a unique solution to the problem* (0.9), (0.10), *having the form*

$$\eta(t) = -\sum_{q=0}^{p} H^q M_0^{-1}(\mathbb{I}-Q)\overset{\circ}{W}{}^{(q+1)}_{\mathbf{K}}(t)+$$
$$+\sum_{j=0}^{m}\left[U_j^{t-\tau_j}\xi_j + L_{1j}^{-1}Q_jNW_{\mathbf{K}}(t) - S_jP_j\int_{\tau_j}^{t} U_j^{s-\tau_j} L_{1j}^{-1} Q_j NW_{\mathbf{K}}(s)ds\right],\ t\in\overline{\mathbb{R}}_+.$$

3. Linear stochastic Oskolkov system

Let $\Omega \subset \mathbb{R}^n$, $n \in \mathbb{N} \setminus \{1\}$, be a bounded domain with boundary $\partial\Omega$ of the class C^∞. In the cylinder $\Omega \times \mathbb{R}$ consider the linear Oskolkov system of equations

$$(1 - æ\nabla^2)v_t = \nu\nabla^2 v - \nabla p + f, \quad \nabla \cdot v = 0. \tag{3.1}$$

Based on the results of points 2 and 3, we will reduce the system (3.1) and the condition (0.3) to the equation (0.9). Following [18], [32], [33], we denote by $\mathbb{H}^2 = (W_2^2)^n$, $\overset{\circ}{\mathbb{H}}{}^1 = (\overset{\circ}{W}{}_2^1)^n$, $\mathbb{L}^2 = (L_2)^n$ space vector-functions $v = (v_1, v_2, \ldots, v_n)$ defined on Ω. Consider the lineal $\mathfrak{L} = \{v \in (C_n^\infty)^n : \nabla \cdot v = 0\}$ of vector-functions, solenoidal and finite in the domain Ω. We denote the closure of $\mathfrak{L}$ with respect to the norm of the space $\mathbb{L}^2$ by $\mathbb{H}_\sigma$. The space $\mathbb{H}_\sigma$ is Hilbert with the scalar product $\langle \cdot, \cdot \rangle$ inherited from $\mathbb{L}^2$; in addition, there is a splitting $\mathbb{L}^2 = \mathbb{H}_\sigma \oplus \mathbb{H}_\pi$, where $\mathbb{H}_\pi$ is the orthogonal complement of $\mathbb{H}_\sigma$. We denote by $\Sigma : \mathbb{L}^2 \to \mathbb{H}_\sigma$ the corresponding orthoprojector. The restriction of the projector Σ to $\mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1$ is a continuous operator, $\Sigma : \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 \to \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1$. Let us therefore represent the space $\mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1$ as a direct sum $\mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 = \mathbb{H}^2_\sigma \oplus \mathbb{H}^2_\pi$, where $\mathbb{H}^2_\sigma = \operatorname{im}\Sigma$, $\mathbb{H}^2_\pi = \ker\Sigma$. There are continuous and dense embeddings $\mathbb{H}^2_\sigma \hookrightarrow \mathbb{H}_\sigma$ and $\mathbb{H}^2_\pi \hookrightarrow \mathbb{H}_\pi$. The space $\mathbb{H}^2_\pi$ consists of vector functions that are equal to zero on $\partial\Omega$ and are gradients of functions from $W_2^3(\Omega)$.

Lemma 3.1. [19]

(i) By the formula $A = (-\nabla^2)^n : \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 \to \mathbb{L}^2$ defines a linear continuous operator with a positive discrete finite multiple spectrum $\sigma(A)$ condensing only to the point $+\infty$, and the mapping $A : \mathbb{H}^2_{\sigma(\pi)} \to \mathbb{H}_{\sigma(\pi)}$ is bijective.

(ii) Formula $B : v \to -\nabla(\nabla \cdot v)$ a linear continuous surjective operator is given $B : \mathbb{H}^2 \cap \overset{\circ}{\mathbb{H}}{}^1 \to \mathbb{H}_\pi$, and $\ker B = \mathbb{H}^2_\sigma$.

Let $\mathfrak{U} = \mathbb{H}^2_\sigma \times \mathbb{H}^2_\pi \times \mathbb{H}_p$, $\mathfrak{F} = \mathbb{H}_\sigma \times \mathbb{H}_\pi \times \mathbb{H}_p$ are real separable Hilbert spaces with an orthonormal basis $\{\varphi_k\}$ and $\{\psi_k\}$, respectively. Moreover $\mathbb{H}_p = \mathbb{H}_\pi$, $A_\lambda = \lambda\mathbb{I} + A$. Let's construct operators

$$L = \begin{pmatrix} \Sigma A_\lambda & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \Pi A_\lambda & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix}, \quad M = \begin{pmatrix} -\nu \sum A & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & -\nu\Pi A & -\ Pi \\ \mathbb{O} & \Pi B & \mathbb{O} \end{pmatrix}.$$

Obviously, $L, M \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$, and $\operatorname{im} L = \mathbb{H}_\sigma \times \mathbb{H}_\pi \times \{0\}$, $\ker L = \{0\} \times \{0\} \times \mathbb{H}_p$.

Let us introduce into consideration a monotone sequence $\mathbf{K} = \{\lambda_k\} \subset \{0\} \cup \mathbb{R}$ such that $\sum\limits_{k=1}^{\infty} \lambda_k^2 < +\infty$. Let us consider Hilbert spaces, which is the completion of the linear hull of *random* $\mathbf{K}$*-variables*

$$\mathbf{U_K L_2} = \left\{ \xi = \sum_{k=1}^{\infty} \lambda_k \xi_k \varphi_k,\ \xi_k \in \mathbf{L_2},\ \varphi_k \in \mathbb{H}^2_\sigma \times \mathbb{H}^2_\pi \times \mathbb{H}_p \right\},$$

$$\mathbf{F_K L_2} = \left\{ \zeta = \sum_{k=1}^{\infty} \mu_k \zeta_k \psi_k,\ \zeta_k \in \mathbf{L_2},\ \psi_k \in \mathbb{H}_\sigma \times \mathbb{H}_\pi \times \mathbb{H}_p \right\}.$$

Lemma 3.2. [19] *For any* $\lambda \in \mathbb{R} \setminus \sigma(A)$, $\nu \in \mathbb{R}_+$ *the operator* M *is* $(L, 1)$*-bounded.*

If we set $N\omega = \mathrm{col}(\Sigma f, \Pi f, 0)$, and $f(t) = \overset{\circ}{W}_{\mathbf{K}}(t)$, then the reduction of the problem (0.3), (3.1) to the equation (0.9) finished.

Let the operators L, M, $N \in \mathcal{L}(\mathbf{U_K L}_2; \mathbf{F_K L}_2)$. Consider a linear stochastic Sobolev type equation (0.9) with multipoint initial-final value condition (0.10).

Let us call the stochastic **K**-process $\eta \in \mathbf{C}^1(\mathbb{R}_+; \mathbf{L}_2)$ (*classical*) *solution of the equation* (0.9), if a.s. all of it trajectories satisfy the equation (0.9) for some **K**-"noise" $\omega \in \mathbf{C}(\mathbb{R}_+; \mathbf{L}_2)$ and all $t \in \mathbb{R}_+$. Solution $\eta = \eta(t)$ to the equation (0.9) let's call *solution to the problem* (0.9), (0.10), if the condition (0.10) is met for some random **K**-variables $\xi_k \in \mathbf{U_K L}_2$, $k = \overline{0, l}$.

Let us find the L-spectrum $\sigma^L(M)$ of the operator M. As is easy to see, the operator $\mu L - M$ is invertible exactly when the operator

$$\Sigma(\mu\lambda\mathbb{I} - (\mu - \nu)A) : \mathbb{H}^2_\sigma \to \mathbb{H}_\sigma$$

is invertible. Let $\tilde{A}$ denote the restriction of the operator A to $\mathbb{H}^2_\sigma$. The spectrum of the operator $\tilde{A} \in \mathcal{L}(\mathbb{H}^2_\sigma; \mathbb{H}_\sigma)$ is positive, discrete, finitely multiple and condenses only to $+\infty$ (Solonnikov – Vorovich – Yudovich theorem [31]). Let $\{\lambda_k\}$ denote the set of eigenvalues of the operator $\tilde{A}$, numbered in non-decreasing order taking into account multiplicity. Then

$$\sigma^L(M) = \left\{ \mu_k = \frac{\nu\lambda_k}{\lambda_k - \lambda} : \lambda_k \in \sigma(\tilde{A}) \setminus \{\lambda\} \right\}.$$

It is clear that for such a set one can select contours $\gamma_j \subset \mathbb{C}$. Let's construct

$$U_j^t = \begin{pmatrix} \sum\limits_{\lambda_k \in \sigma_j^L(M)} e^{\lambda_k t} \langle \cdot, \varphi_k \rangle_\sigma \varphi_k & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix}, j = \overline{0, m}.$$

It follows from Lemma 3.2 that under the conditions of this lemma the condition (0.8) is satisfied.

Theorem 3.3. *Let the operators* L *and* M *be defined as in Lemma 3.2. Then for any* $\tau_j \in \mathbb{R}_+$, $j = \overline{1, m}$, *operator* $N \in \mathcal{L}(\mathfrak{U}; \mathfrak{F})$, *monotonic sequence* $\mathbf{K} \subset \{\lambda_k\}$ *such that* $\sum\limits_{k=1}^{\infty} \lambda_k^2 < +\infty$, **K**- *"noise"* $\omega = \omega(t)$ *such that* $(\mathbb{I} - Q)Nw \in \mathbf{C}^{p+1}(\mathbb{R}_+; \mathbf{U_K L}_2)$ *and* $QNw \in \mathbf{C}(\mathbb{R}_+; \mathbf{U_K L}_2)$, $\omega(t) = \overset{\circ}{W}_{\mathbf{K}}(t)$ *and random* **K***-variables* $\xi_j \in \mathbf{U_K L}_2$, $j = \overline{0, m}$, *independent of* ω, *there is a unique solution* $\eta \in \mathbf{C}^1(\mathbb{R}_+; \mathbf{U_K L}_2)$, *problem* (0.9), (0.10), *having the form*

$$\eta(t) = \sum_{j=0}^{m} \left[U_j^{t-\tau_j} \xi_j + L_{1j}^{-1} Q_j N W_{\mathbf{K}}(t) - S_j P_j \int_{\tau_j}^{t} U_j^{s-\tau_j} L_{1j}^{-1} Q_j N W_{\mathbf{K}}(s) ds \right] -$$

$$- \sum_{q=0}^{p} H^q M_0^{-1} (\mathbb{I} - Q) \overset{\circ}{W}{}_{\mathbf{K}}^{(q+1)}(t), \; t \in \overline{\mathbb{R}}_+.$$

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References

1. Oskolkov, A.P.: Solvability in the large of the first boundary value problem for a certain quasilinear third order system that is encountered in the study of the motion of a viscous fluid, *Zap. Naucn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI)* **27** (1972) 145–160.
2. Oskolkov, A.P.: Nonlocal problems for some class nonlinear operator equations arising in the theory Sobolev type equations, *Journal of Soviet Mathematics* **64** (1) (1993) 724–736.
3. Sviridyuk, G. A.: On the manifold of solutions of a problem on the dynamics of an incompressible viscoelastic fluid, *Differentsial'nye Uravneniya*, **24** (10) (1988) 1832–1834. (in Russian)
4. Astarita, G. and Marrucci, G.: *Principles of non-newtonion fluid mechanics.* McGraw-Hill Book Company (UK) Ltd., 1974.
5. Vinogradov, G.V. and Malkin, A.Ya.: *Reologiya Polimerov (Rheology of Polymers)*, Moscow, Khimiya, 1977. (in Russian)
6. Christensen, R.M.: *Theory of Viscoelasticity. An Introduction*, New York and London, Academic Press, 1971. https://doi.org/10.1016/B978-0-12-174252-2.X5001-7
7. Ladyzhenskaya, O.A.: *The Mathematical Theory of Viscous Incompressible Flow*, New-Yourk, London, Paris, Gordon and Breach, Science Publ., 1969.
8. Temam, R.: *Navier—Stokes Equations. Theory and Numerical Analysis*, Amsterdam, N.-Y., Oxford, North Holland Publ. Co., 1979.
9. Pavlovskii, V. A.: To the Question of Theoretical Description of Weak Aqueous Solutions of Polymers, *Dokl. Akad. Nauk SSSR* **200** (4) (1971) 809–812. (in Russian)
10. Voitkunskii, Ya.I., Amfilokhiev, V.B. and Pavlovskii, V. A.: Uravneniya dvizheniya zhidkosti s uchetom ee relaksatsionnykh svoistv, *Tr. Leningradsk. ordena Lenina korablestroitelnogo instituta* **69** (1970) 16–24. (in Russian)
11. Amfilokhiev, V.B., Voitkunskii, Ya.I., Mazaeva, N.P. and Khodorkovskii, Ya.S.: Flows of polymeric solutions in presence of convective accelerations, *Tr. Leningradsk. ordena Lenina korablestroitelnogo instituta* **96** (1975) 3–9. (in Russian)
12. Sviridyuk, G. A. and Sukacheva, T. G.: Zametki o lineinykh modelyakh vyazkouprugikh sred, *Vestnik Chelyabinskogo Gosudarstvennogo Universiteta. Matematika, Mekhanika, Informatika*, **3** (1996) 135–147 (in Russian)
13. Oskolkov, A.P.: Some nonstationary linear and quasilinear systems occurring in the investigation of the motion of viscous fluids, *Journal of Soviet Mathematics* **10** (2) (1978), 299–335
14. Oskolkov, A.P.: Initial-boundary value problems for equations of motion of Kelvin – Voight fluids and Oldroyd fluids, *Proceedings of the Steklov Institute of Mathematics*, **179** (1989) 137–182.
15. Oskolkov, A.P.: To the stability theory for the solutions of the semilinear dissipative Sobolev type equations, *Journal of Mathematical Sciences*, **77** (3) (1995) 3225–3231. DOI: https://doi.org/10.1007/BF02364715
16. Oskolkov, A.P., Achmatov, M.M. and Shadiev R.D.: Nonlocal problems for the equations of filtration of nonnewtonian fluids in porous media, *Journal of Soviet Mathematics* **62** (5) (1992), 2992–3004.
17. Kotsiolis, A.A., Oskolkov, A.P. and Shadiev R.D.: Nonlocal problems for some class nonlinear dissipative Sobolev type equations, *Journal of Mathematical Sciences*, **77** (2) (1995) 3076–3089.
18. Sviridyuk, G. A.: On a model of the dynamics of a weakly compressible viscoelastic fluid, *Russian Math. (Iz. VUZ)*, **38** (1) (1994) 59–68.
19. Sviridyuk, G. A.: On the general theory of operator semigroups, *Russian Math. Surveys*, **49** (4) (1994) 45–74.

20. Zagrebina S.A. and Konkina A.S. The non-classical models of mathematical physics the multipoint initial-final value condition, *Bulletin of the South Ural State University. Series "Mathematical Modelling, Programming & Computer Software"* **15** (1) (2022) 60–83. DOI: https://doi.org/10.14529/mmp220104 (in Russian)
21. Nelson, E. *Dynamical Theory of Brownian Motion*, Princeton, Princeton University Press, 1967.
22. Gliklikh, Yu.E. *Global and Stochastic Analysis with Applications to Mathematical Physics*, London, Dordrecht, Heidelberg, N.-Y., Springer, 2011. DOI: 10.1007/978-0-85729-163-9
23. Sviridyuk, G.A. and Manakova, N.A.: The Dynamical Models of Sobolev Type with Showolter–Sidorov Condition and Additive "Noise", *Bulletin of the South Ural State University. Series "Mathematical Modelling, Programming & Computer Software"* **7** (1) (2014) 90–103. DOI: 10.14529/mmp140108 (in Russian)
24. Gliklikh Yu.E. and Zalygaeva M.E.: Non-Newtonian fluids and stochastic analysis on the groups of diffeomorphisms, *Applicable Analysis*, **94** (6) (2014) 1116–1127.
25. Favini, A., Sviridyuk, G.A. and Manakova, N.A.: Linear Sobolev Type Equations with Relatively p-Sectorial Operators in Space of "noises", *Abstract and Applied Analysis* **1** (2015) 1–8. DOI: 10.1155/2015/697410
26. Favini, A., Sviridyuk, G.A. and Sagadeeva, M.A.: Linear Sobolev Type equations with relatively p-Radial Operators in Space of "Noises", *Mediterranean Journal of Mathematics* **13** (6) (2016) 4607–4621. DOI: 10.1007/s00009-016-0765-x
27. Favini, A., Sviridyuk, G.A. and Zamishlyaeva, A.A.: One class of Sobolev type equations of higher order with additive white noise", *Communications on Pure and Applied Analysis* **15** (1) (2016) 185–196. DOI: 10.3934/cpaa.2016.15.185
28. Favini, A., Zagrebina, S.A. and Sviridyuk, G.A.: Multipoint initial-final value problems for dynamical Sobolev-type equations in the space of noises, *Electronic Journal of Differential Equations* **128** (2018) ID article 128, 1–10. URL: https://ejde.math.txstate.edu/
29. Favini, A., Zagrebina, S.A. and Sviridyuk, G.A.: The Multipoint Initial–Final Value Condition for the Hoff Equations on Geometrical Graph in Spaces of K -"noises", *Mediterranean Journal of Mathematicsthis* **19** (2) (2022) ID article 53 1–10.
30. Gliklikh, Yu.E. and Mashkov, E.Yu.: Stochastic Leontieff type equations and mean derivatives of stochastic processes, *Bulletin of the South Ural State University. Series "Mathematical Modelling, Programming & Computer Software"* **6** (2) (2013) 25–39. (in Russian)
31. Zagrebina, S., Sukacheva, T. and Sviridyuk, G.: The multipoint initial-final value problems for linear Sobolev-type equations with relatively p-sectorial operator and additive noise, *Global and Stochastic Analysis* **5** (2) (2018) 129–143.
32. Sukacheva, T.G.: *Extended phase spaces of Oskolkov models. Theory of non-autonomous equations of Sobolev type and its applications: monograph*, Saarbrücken, LAMBERT Academic Publishing, 2011. (in Russian)
33. Matveeva, O.P. and Sukacheva, T.G.: *Mathematical models of viscoelastic incompressible liquids of non-zero order: monograph*, Chelyabinsk, Publishing Center of SUSU, 2014. – 101 p. (in Russian)

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