Global and Stochastic Analysis
Vol. 11 No. 4 (September, 2024)

Received: 16th June 2024 Revised: 20th July 2024 Accepted: 16th August 2024

FURTHER RESULTS ON FINITE PRIME DISTANCE GRAPHS

B. SRIPATHY, LEENA ROSALIND MARY G, NAGALAKSHMI VALLABHANENI, P. SELVARAJU, RAM DAYAL, A. PARTHIBAN*, AND ABHINAV SUDHAKAR DUBEY**

Abstract. Graph labeling is an assignment of labels to the nodes/lines or both of a graph G_α subject to a few norms. The field of graph theory, particularly graph labeling, plays a vital role in various fields such as $x-$ray crystallography, radar, astronomy, etc. A graph G_γ is a prime distance graph if its nodes can be assigned with unique integers such that for any two adjacent nodes, the positive difference of their labels is a prime number. Laison et al. [9] have raised the following questions. (1) Is there a family of graphs which are prime distance graphs if and only if Goldbach's conjecture is true? (2) What other families of graphs are PDGs? Parthiban et al. answered these questions to some extend [13]. In continuation, in this paper these questions are answered further by establishing prime distance labeling of certain graphs using Goldbach's conjecture and the Twin prime conjecture, besides characterizing certain classes of graphs in terms of prime distance labeling and formulating interesting conjectures.

1. Introduction

The graphs considered in the present study are "simple, finite, undirected, and connected". As usual, let G, P_n, C_n, K_n denote a graph, path, cycle, and complete graph on n nodes, respectively. For other graph theoretic terminologies, refer [15] and number theoretic concepts, refer [3]. Let Z and P represent the set of all integers and primes, respectively. The concept of prime distance graph (PDG) was introduced by Eggleton et al. [4, 5]. "For any set D_k of positive integers, they considered the distance graph $Z(D_k)$ as the graph with node set Z and a line between integers s and t if and only if $|s-t| \in D_k$. The prime distance graph $Z(P)$ is the distance graph with $D_k = P$". One can notice that the PDG is infinite. This paper deals with finite subgraphs of $Z(P)$.

Laison et al. [9] gave the notion of finite prime distance graph. A graph G_α is a PDG if there exists a one-to-one labeling of its nodes $t : V(G_\alpha) \to Z$ such that for any two adjacent nodes x_1 and x_2, the integer $|t(x_1) - t(x_2)|$ is a prime and t is called a prime distance labeling (PDL) of G_α.

Laison et al. have showed very interesting and remarkable results using various number theoretic statements and results in [9] and also raised the following questions. (1) Is there a family of graphs which are PDGs if and only if Goldbach's conjecture is true? (2) What other families of graphs are PDGs?. Parthiban et al. answered these questions to some extend [1, 11, 13]. In continuation of [13], in this paper, we extend the results further and explore a few more classes of PDGs.

2000 *Mathematics Subject Classification.* 05C78, 05C38.

Key words and phrases. Finite Prime Distance Graph, Goldbach's Conjecture, Twin Prime Conjecture, Limit Graph, Double and Strong Double Graph.

*Corresponding author.
**Undergraduate student.

2. Some Important Known Results

This section recollects some results concerning PDL for the sake of completeness and understanding.

Theorem 2.1. [9] *"Every subgraph of a PDG is PDG".*

Lemma 2.2. [9] *"Any graph G_f with $\chi(G_f) \geq 5$ cannot have PDL".*

Conjecture 2.3. ***(Goldbach's)***[9, 8] *Any even number $2r > 2$ can be written as a sum of two primes.*

Conjecture 2.4. ***("Twin Prime Conjecture (TPC)")***[9, 8] *"There are infinitely many pairs of primes whose difference is 2".*

Lemma 2.5. [11] *"If S is a subgraph of H with no PDL, then H cannot have PDL".*

Theorem 2.6. [13] *"If a graph G_r is formed from a cycle C_r by duplicating an arbitrary node by a node, then G_r allows PDL for all $r \geq 6$ if and only if Goldbach's conjecture is true."*

Theorem 2.7. [13] *"The triangular book with book marks, $B_n^{(3)}$, allows PDL $\forall n \in Z^+$ if and only if the TPC is true."*

Theorem 2.8. [13] *"The jewel graph J_n allows PDL if and only if the TPC is true."*

Theorem 2.9. [13] *"The pizza graph Pz_r allows PDL $\forall r \geq 3$ if the TPC is true."*

Theorem 2.10. [13] *"The generalized Jahangir graph, $J_{m,k}$, allows PDL $\forall m \geq 3, k \geq 1$."*

Theorem 2.11. [9] *"Every bipartite graph allows PDL."*

3. Main Results

In this section, PDL of some new classes of graphs are derived.

Theorem 3.1. *The graph obtained by gluing finite copies of $K_n, 1 \leq n \leq 4$ by a line allows PDL.*

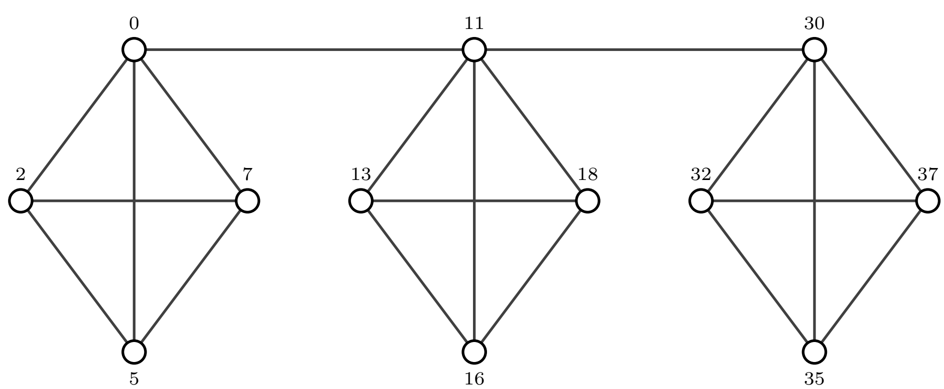

Figure 1. PDL of a graph formed by gluing three copies of K_4 by a line

Proof. One can obtain the PDL of the graph formed by joining finite copies of K_r by a line for $1 \leq r \leq 3$. So, we consider, the graph G_α, constructed by joining finite, say r, copies of K_4 by a line. Obviously, $|V(G_\alpha)| = 4r$ and $|E(G_\alpha)| = 7r - 1$. Further, let $\{v_i^j : 1 \leq i \leq 4; 1 \leq j \leq r\}$ be the nodes of G_α. Define an injective map $h_\alpha : V(G_\alpha) \to Z$ as follows: WLG, let $h_\alpha(v_1^1) = 0$, $h_\alpha(v_2^1) = 2$, $h_\alpha(v_3^1) = 5$, and $h_\alpha(v_4^1) = 7$. Now, let p_1 be the sufficiently large prime number than the already used labels. Then, $h_\alpha(v_i^2)$=$h_\alpha(v_i^1) + p_1$, $1 \leq i \leq 4$. Similarly, let p_2 be the sufficiently large prime number than the used labels. Then, $h_\alpha(v_i^3)$= $h_\alpha(v_i^1) + p_2$: $1 \leq i \leq 4$. Continuing thus, for the r^{th} copy of K_4, let p_{r-2} be the sufficiently large prime than the used labels. Then, $h_\alpha(v_i^r)$=$h_\alpha(v_i^{r-1}) + p_{r-2}$. One can verify that h_α is the required PDL of G_α. (see, Figure 1) □

Theorem 3.2. *The graph obtained by gluing a finite copies of K_r by a line does not allow PDL for $r \geq 5$.*

Proof. The proof directly follows from Lemma 2.2 and Lemma 2.5. □

Definition 3.3. [7] "Let G_x and G_y be two graphs with no node in common. The join of G_x and G_y, denoted by G_x+G_y, defined to be the graph as follows: $V(G_x+G_y) = V(G_x) \cup V(G_y), E(G_x+G_y) = E(G_x) \cup E(G_y) \cup \{t_1t_2 : t_1 \in V(G_x), t_2 \in V(G_y)\}$."

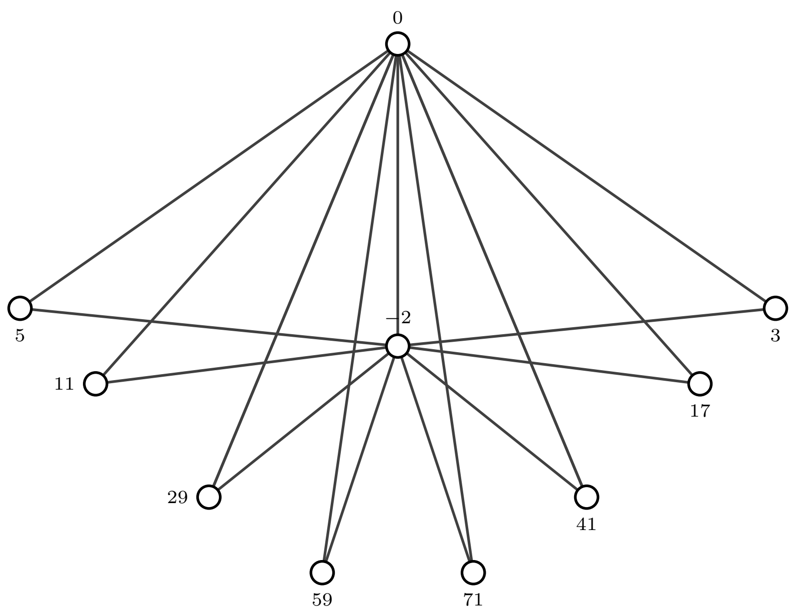

Figure 2. PDL of $K_1 + K_{1,8}$

Theorem 3.4. *The graph $K_1 + K_{1,n}$ has a PDL if and only if the TPC is true.*

Proof. Let $\{x_0, x_1, x_2, ..., x_n\}$ be the nodes of $K_{1,n}$ with central node x_0 and $\{t\}$ be the node of K_1 so the lines of $K_1 + K_{1,n}$ are $\{tx_0, tx_i, x_0x_i, i = 1, 2, .., n\}$. Here, $|V(K_1 + K_{1,n})| = n + 2$ and $|E(K_1 + K_{1,n})| = 2n + 1$. First suppose that $K_1 + K_{1,n}$ has a PDL for an arbitrary $n \geq 1$, and consider one such PDL of $K_1 + K_{1,n}$. WLG, suppose that t and x_0 are assigned with 0 and -2,

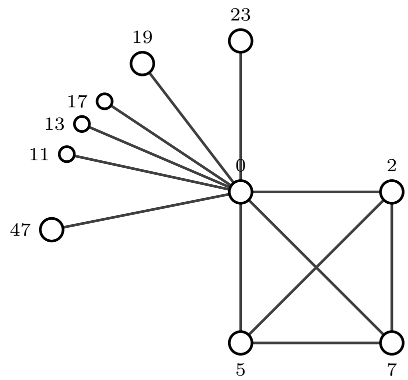

Figure 3. PDL of $K_4 + S_6$

respectively. Note that the remaining nodes cannot be of even labels. In each of these K_3, since both even-labeled nodes are connected to $x_i; 1 \leq i \leq n$, their labels must be odd, more specifically, primes. Since their difference is a prime number, so each x_i is assigned with a prime number, so that their difference with t and x_0 produce twin primes. i.e., if $K_1 + K_{1,n}$ is a PDG, there are exactly n twin primes. Hence, if all $K_1 + K_{1,n}$ are PDG, then the TPC is true.

Conversely, if the TPC is true, then assigning t with 0, x_0 with -2, and x_i of each K_3 with a prime p (where $p + 2$ is also prime) is a PDL of $K_1 + K_{1,n}$. (See, Figure 2) □

Definition 3.5. "The graph $K_r + S_r$ is obtained by attaching to the center of S_r to a node of K_r".

Lemma 3.6. *The graph $K_n + S_m$ allows PDL for $1 \leq n \leq 4$ and any $m \in N$.*

Proof. The proof is simple for the case of $n = 1, 2, 3$. So, consider the case of $n = 4$. Thus, the graph obtained is $K_4 + S_m, \forall m \in N$ with $V(K_4) = v_i : 1 \leq i \leq 4$ and $V(S_m) = u_j : 0 \leq j \leq m$. WLG, let the center node of S_m, say u_0, be merged with $v_1 \in K_4$. Define a one-one function $h_\alpha : V(K_4 + S_m) \to Z$ as follows: WLG, let $h_\alpha(v_1) = 0, h_\alpha(v_2) = 2, h_\alpha(v_3) = 5$, and $h_\alpha(v_4) = 7$. Then, $h_\alpha(u_i) = p_i$; for $1 \leq i \leq m$, where $p_i^\prime s$ are sufficiently large primes. One can easily verify that h_α is the PDL of $K_4 + S_m$. (see, Figure 3) □

Theorem 3.7. *The graph $K_n + S_m$ does not allow PDL for $n \geq 5$ and $m \in N$.*

Definition 3.8. [14] The Cartesian product of graphs G_z and H_z is the graph $G_z \times H_z$ with node set $V(G_z \times H_z) = V(G_z) \times V(H_z)$ and line set $E(G_z \times H_z) = \{(g,h)(g',h')|gg' \in E(G_z)$ and $h = h'$, or $hh' \in E(H)$ and $g = g'\}$.

Theorem 3.9. *The graph $C_n \times P_m$ allows PDL for any $n \geq 3$ and $m \in N$ if Goldbach's conjecture is true.*

Proof. Let C_n be the given cycle and P_m be the path on n and m nodes, respectively. Obtain $C_n \times P_m$ with $V(C_n \times P_m) = \{v_i^j$; for $1 \leq i \leq n$ and $1 \leq j \leq m\}$. Define $h_\beta : V(C_n \times P_m) \to Z$ as follows: WLG, let $h_\beta(v_i^1) = 2(i-1); 1 \leq i \leq n-1$. Now, if the Goldbach's conjecture is true, then $h_\beta(v_{n-1}^1) = p*_1 + p*_2$, where $p*_1$ and $p*_2$ are primes. Then, let $h_\beta(v_n^1) = p*_1$ or $p*_2$ so that $|h_\beta(v_{n-1}^1) - h_\beta(v_n^1)|$ is a prime. Also, let p_1 be the first prime greater than the used labels. Then, $h_\beta(v_i^2) = h_\beta(v_i^1) + p_1$ for $1 \leq i \leq n$. Similarly, let p_2 be the first prime greater than the used labels.

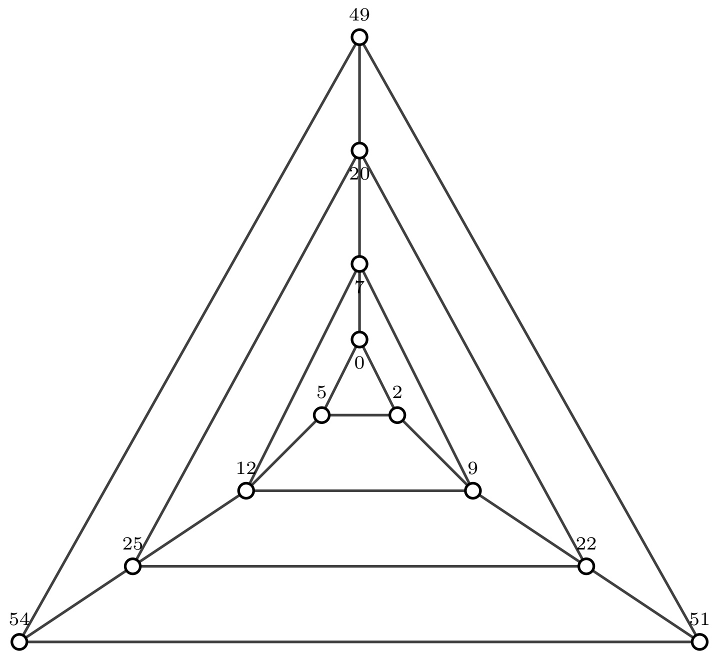

Figure 4. PDL of $C_3 \times P_4$

Then, $h_\beta(v_i^3) = h_\beta(v_i^1) + p_2$ for $1 \leq i \leq n$. Continuing thus, let p_{m-1} be the first prime greater than the used labels. Then, $h_\beta(v_i^m) = h_\beta(v_i^1) + p_{m-1}$ for $1 \leq i \leq n$. One can check that h_β is the PDL of $C_n \times P_m$. (see, Figure 4) □

Conjecture 3.10. *The graph obtained by replacing every line of a star graph $K_{1,n}$ by the bipartite graph $K_{1,n,1}$ does not allow PDL for $n \geq 3$.*

Theorem 3.11. *The line graph $L(G_\alpha)$ of G_α is the graph whose nodes are the lines of G_α, with $ef \in E(L(G_\alpha))$ where $e = uv$ and $f = vw$ in G_α.*

Definition 3.12. The armed crown ACr_n is a graph in which P_2 is attached at each node of C_n by a line.

Theorem 3.13. *The graph $L(ACr_n)$ allows PDL if Goldbach's conjecture is true.*

Proof. For ACr_n, let $x_1, x_2, ..., x_n$ be the lines of C_n and $x'_1, x'_2, ..., x'_n$ corresponding to paths and x''_1,x''_2, ..., x''_n be the lines joining path and C_n. Then, $V(L(ACr_n)) = \{x_1, x_2, ..., x_n, x'_1, x'_2, ..., x'_n, x''_1, x''_2, ..., x''_n\}$. Here, $|V(L(ACr_n))| = 3n$ and $|E(L(ACr_n))| = 4n$. Define $h_\beta : V(L(ACr_n)) \to Z$ as follows: WLG, let $h_\beta(v_i) = 2(i-1); 1 \leq i \leq n-1$. Now, if Goldbach's conjecture is true, then $h_\beta(v_{n-1}) = p_1 + p_2$, where p_1 and p_2 are primes. Then, $h_\beta(v_n) = p_1$ or p_2 so that $|h_\beta(v_n) - h_\beta(v_{n-1})|$ is prime and $|h_\beta(v_n) - h_\beta(v_1)|$ also prime. Then, let p_{r_1} be the sufficiently large unused prime number. Then, $h_\beta(u_1) = p_{r1}, h_\beta(u_i) = h_\beta(u_{i-1}) + 2$ for $2 \leq i \leq n-2$; $h_\beta(u_{n-1}) = h_\beta(v_{n-1}) + 2$; $h_\beta(u_n) = p_2$. Finally, $h_\beta(w_i) = h_\beta(u_i) + p_i$ where p'_is are sufficiently large prime numbers. Hence the proof. (see, Figure 5) □

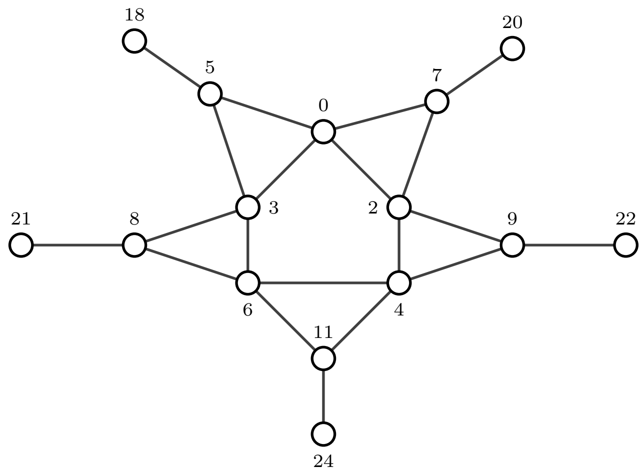

Figure 5. PDL of $L(ACr_5)$

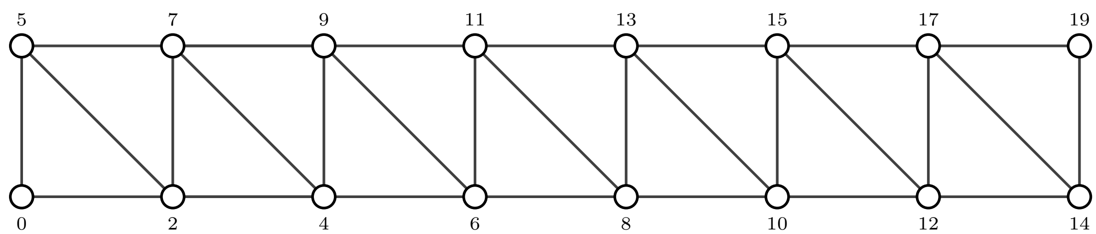

Figure 6. PDL of $TB(8)$

Definition 3.14. Let $L_k = P_k \times P_2 (k \geq 2)$ be the ladder graph with node set x_i and $y_i, 1 \leq i \leq n$. The triangular belt, $TB(k)$, is fixed from L_k by adding the lines $x_i y_{i+1} \forall 1 \leq i \leq k-1$.

Theorem 3.15. *$TB(k)$ allows PDL for all $k \in N$.*

Proof. Let $TB(k)$ be the given triangular belt with $V(TB(k))$=$V_1 \cup V_2$; where $V_1 = \{u_i; 1 \leq i \leq k\}$ and $V_2 = \{v_i; 1 \leq i \leq k\}$. Define $h_\gamma : V(TB(k)) \to Z$ as follows: WLG, let $h_\gamma(u_i) = 2(i-1); 1 \leq i \leq k$ and $h_\gamma(v_1) = h_\gamma(u_1) + 5$. Then, $h_\gamma(v_i) = h_\gamma(v_{i-1}) + 2; \forall 2 \leq i \leq k$. Thus, h_γ is the PDL of $TB(k)$. (see, Figure 6) □

Definition 3.16. The alternate triangular belt, $ATB(k)$, is derived from L_k by adding the lines $x_{2i+1} y_{2i+2}$ $\forall$ $i = 1, 2, ..., k-1$ and $x_{2i} y_{2i+1}$ $\forall$ $i = 1, ..., k-1$.

Theorem 3.17. *$ATB(k)$ allows PDL $\forall k \in N$.*

Proof. The proof is same as given in the above theorem. □

3.1. PDL of Double and Strong Double Graphs. In this section, PDL of double and strong double graphs of some graphs are derived.

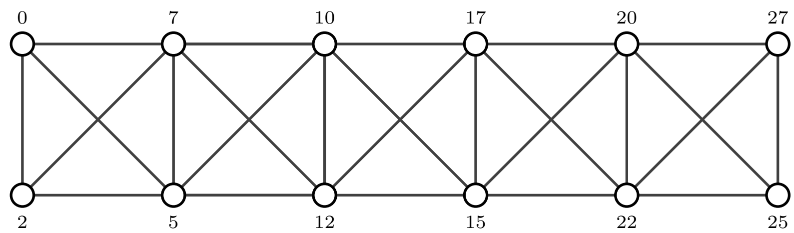

Figure 7. PDL of $SD(P_6)$

Definition 3.18. [10] "For a graph G_x, the double graph $D[G_x]$ is a graph formed by taking two copies of G_x and joining each node in one copy with the neighbors of corresponding node in another copy".

Lemma 3.19. [6] *For any graph H_x, H_x is bipartite if and only if $D[H_x]$ is bipartite.*

Theorem 3.20. *For any bipartite graph G_y, $D[G_y]$ allows PDL.*

Proof. The proof directly follows from Theorem 2.11 and Lemma 3.19. □

Lemma 3.21. [6] *For any graph G_r, $\chi(D[G_r]) = \chi(G_r)$.*

Theorem 3.22. *For any graph G_z with $\chi(G_z) \geq 5$, $D[G_z]$ does not allow PDL.*

Proof. The proof follows from Lemma 3.21 and the fact that the chromatic number of any PDG is less than or equal to 4. □

Definition 3.23. [6] "The lexicographic product of two graphs G_x and H_y is the graph $G_x \circ H_y$ with $V(G_x) \times V(H_y)$ as node set and with adjacency defined by (x_1, y_1) adj(x_2, y_2) if and only if $x_1 = x_2$ and y_1 adj y_2 in H_y or x_1 adj x_2 in G_x. The graph $G_x \circ H_y$ can be constructed from G_x substituting to each node x of G_x a copy H_s of H_y and joining every node of H_s with every node of H_t whenever s and t are adjacent in G_x."

Lemma 3.24. [6] *For any graph G_t on t nodes, $D[G_t] = G_t \circ N_2$ and $D[G_t]$ is $t-$partite, where N_k is the graph on k nodes without lines.*

Theorem 3.25. *For any graph G_z on z nodes, $D[G_z] = G_z \circ N_k$ does not allow PDL for $k \geq 5$.*

Definition 3.26. [10] "The strong double graph $SD(G_s)$ of G_s, is formed by taking two graphs and joining the closed neighborhood of each node in one graph to the adjacent node in the other graph".

Theorem 3.27. *$SD(P_n)$ allows prime distance labeling $\forall n \geq 1$.*

Proof. Let P_n be the given path on n nodes, namely $v_1, v_2, ..., v_n$. Take two copies of P_n and obtain $SD(P_n)$ as defined above.

Define an injective function $h_\gamma : V(SD(P_n)) \rightarrow Z$ as follows: WLG, let $h_\gamma(v_1) = 0, h_\gamma(v_2) = 7, h_\gamma(u_1) = 2$, and $h_\gamma(u_2) = 5$. Then, $h_\gamma(v_3)= h_\gamma(u_2) + 5$, $h_\gamma(u_3)= h_\gamma(v_2) + 5$, ..., $h_\gamma(v_k)= h_\gamma(u_{k-1}) + 5$, and $h_\gamma(u_k)= h_\gamma(v_{k-1}) + 5$. One can see that h_γ is the PDL of $SD(P_n)$. (see, Figure 7) □

3.2. PDL of Generalized Cone Graphs. In this section, we completely characterize the PDL of generalized cone graphs.

Definition 3.28. A generalized cone graph (GCG), denoted by $C_s + I_t$, is the join of C_s and an independent set I_t, where $s \geq 3$ and $t \geq 0$.

Theorem 3.29. *$C_s + I_t$ allows PDL for $s \geq 3$ and $t = 0$.*

Proof. The proof follows from the fact that the resultant graph is a cycle and every cycle admits PDL. □

Theorem 3.30. *$C_s + I_t$ allows PDL for $3 \leq s \leq 8$ and $t = 1$.*

Proof. The proof follows from the fact that the resultant graph is a wheel and every wheel $W_n, n \leq 9$ allows PDL. □

Theorem 3.31. *$C_s + I_t$ does not allow PDL for $s \geq 9$ and $t = 1$.*

Proof. The proof follows from the fact that the resultant graph is a wheel and every wheel $W_n, n \geq 10$ does not allow PDL. □

Theorem 3.32. *$C_s + I_t$ does not allow PDL for $s \geq 3$ and $t \geq 2$.*

Proof. Let $C_s + I_t$ be the given GCG on $s + t$ nodes, where $s \geq 3$ and $t \geq 2$. For the sake of discussion, consider the case of $s = 3$ and $t = 2$. The proof now easily follows from the fact that the resultant graph is $K_5 - e$ and one can see that the graph does not admit PDL. The other cases when $s \geq 4$ and $t \geq 3$ can be dealt in the same way. □

3.3. PDL of the Limit Graph of a Graph. In this section, we completely characterize the PDL of limit graph of the given graph. In 2024, B. Akhil et al. [2] introduced the concept of finding the limit graph of a given graph. They also categorized graphs based on whether or not they have a unique limit graph.

Definition 3.33. [2] "Let $G_z = (V, E)$ be a connected graph on z nodes, $z \geq 2$ and $S \subset V(G_z)$. Let $H_z = <S>$ be a connected subgraph of G_z with minimal order and size such that the open neighborhood of S, $N(S) = V(G_z)$. This H_z is said to be the limit graph of G_z and is represented by $lim(G_z)$."

One can see that every graph has at least one limit graph and $lim(G)$ need not be unique.

Theorem 3.34. *The limit graph of any finite prime distance graph is a PDG.*

Now we propose the following conjecture.

Conjecture 3.35. *If G is any finite graph possibly except K_n for large n, then $lim(G)$ allows PDL.*

3.4. Distinct Prime Distance Labeling of Graphs. In this section, we propose an interesting conjecture.

Definition 3.36. [12] "A PDL of a graph G_w is distinct if the absolute differences of the integer labels of adjacent nodes are distinct prime numbers."

Conjecture 3.37. *Almost all graphs are not distinct prime distance graphs.*

This conjecture is proposed by considering the very difficult conditions of the definition. We believe that except a few classes of graphs, almost all other graphs may not admit distinct PDL.

4. Open problems

In addition to the conjectures, we pose the following questions.

(1) For any graph G_s with PDL, do $D[G_s]$ and $SD[G_s]$ also allow PDL?
(2) What are the classes of graphs G_i such that $D[G_i]$ and $SD[G_i]$ allow PDL?
(3) What are the classes of graphs G_i such that $K_1 + G_i$ admit or do not admit PDL?

Conclusion. In this article, the questions raised by Laison et al. [9] are answered to some extend along with establishing PDL for a few notable graphs using Goldbach's conjecture and the Twin prime conjecture. The PDL of certain families of graphs are characterized. So this article may serve as a tool to either completely or partially characterize prime distance graphs. Finally, a few interesting conjectures and open problems are also formulated for the future study.

Acknowledgment. Authors would like to thank the anonymous reviewers for their valuable suggestions.

References

1. Ajaz Ahmad Pir, Tabasum Mushtaq, and Parthiban, A.: 2-Odd Labeling of Graphs Using Certain Number Theoretic Concepts and Graph Operations, *Mathematics and Statistics* **10** (2023) 875-883.
2. Akhil, B., Roy John, Manju, V. N., and Suresh Singh, G.: ARMS-Product and Limit Graph of Graphs, *Global and Stochastic Analysis* **11** (2024) 47-59.
3. Burton, D.M.: *Elementary Number Theory*, McGraw Hill LLC, 2010.
4. Eggleton, R.B., Erdos, P., and Skilton, D.K.: Colouring Prime Distance Graphs, *Graphs and Combinatorics* **6** (1990) 17-32.
5. Eggleton, R.B., Erdos, P., and Skilton, D.K.: Colouring the Real Line, *J. Combin. Theory Ser. B* **39** (1985) 86-100.
6. Emanuele Munarini, Claudio Perelli Cippo, Andrea Scagliola, and Norma Zagaglia Salvi: Double Graphs, *Discrete Mathematics* **308** (2008) 242-254.
7. Gross, J., Yellen, J.: *Graph Theory and Its Applications*, CRC Press, London, 1999.
8. Kenneth, R.H.: *Discrete Mathematics and Its Applications*, CRC Press, 1999.
9. Laison, J.D., Starr, C., and Walker, A.: Finite Prime Distance Graphs and 2-odd Graphs, *Discrete Mathematics* **313** (2013) 2281-2291.
10. Muhammad Imran and Shehnaz Akhter: Degree-based Topological Indices of Double Graphs and Strong Double Graphs, *Discrete Mathematics, Algorithms and Applications* **09** (2017) 1750066.
11. Parthiban, A., David, N.G: On Prime Distance Labeling of Graphs, In: S. Arumugam, Bagga, J., Beineke, Panda, B., (eds) Theoretical Computer Science and Discrete Mathematics. ICTCSDM 2016. *Lecture Notes in Computer Science* **10398**, 2017.
12. Parthiban, A., Gnanamalar David, N., and Nagar, A.K.: Distinct Prime Distance Labeling of Certain Graphs, In: Nagar, A., Deep, K., Bansal, J., Das, K. (eds) Soft Computing for Problem Solving 2019. *Advances in Intelligent Systems and Computing* **1138** Springer, Singapore, 2020.
13. Ram Dayal, Parthiban, A., and Selvaraju, P.: On Questions Concerning Finite Prime Distance Graphs, *Mathematics and Statistics* **12** (2024) 229-233.
14. Vizing, V. G.: The Cartesian Product of Graphs, *Vycisl. Sistemy*, **9** (1963) 30–43.
15. West, D.B.: *Introduction to Graph Theory*, Prentice Hall, 2001.

B. Sripathy: Department of Mathematics, Vellore Institue of Technology, Vellore, Tamil Nadu- 632 014, India
Email address: sripathy.b@vit.ac.in

G. Leena Rosalind Mary: Department of Mathematics, Madanapalle Institute of Technology & Science, Madanapalle, Andhra Pradesh-517 325, India
Email address: leenarosalindg@mits.ac.in

Nagalakshmi Vallabhaneni: School of Computer Science and Information Systems, Vellore Institute of Technology, Vellore, Tamil Nadu-632 014, India
Email address: nagalakshmi.v@vit.ac.in

P. Selvaraju: Department of Computer Science and Engineering, Saveetha School of Engineering, Chennai, Tamil Nadu-602 105, India
Email address: pselvar@yahoo.com

Ram Dayal: Department of Mathematics, Government Degree College Marh, Jammu, Jammu and Kashmir-181 206, India
Email address: ramdayalmath@gmail.com

A. Parthiban: Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu-632 014, India
Email address: parthiban.a@vit.ac.in

Abhinav Sudhakar Dubey: School of Computer Science and Information Systems, Vellore Institute of Technology, Vellore, Tamil Nadu-632 014, India
Email address: dubey.abhinav2003@gmail.com