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ON HOPF LIGHTLIKE HYPER SURFACES OF INDEFINITE COSYMPLETIC MANIFOLD

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Abstract

The object of present paper is to study the properties of Hopf lightlike hypersurfaces of indefinite cosympletic manifold.

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1. Introduction

An (l, m)-type connection [1] is defined as linear connection with $\bar{\nabla}$, $\bar{T}$ holds

$$(\bar{\nabla}_A \bar{g})(B, C) = l\{\theta(B)\bar{g}(A, C) + \theta(C)\bar{g}(A, B)\} - m\{\theta(B)\bar{g}(JA, C) + \theta(C)\bar{g}(JA, B)\} \quad (1.1)$$

$$\bar{T}(A, B) = l\{\theta(B)A - \theta(A)B)\} + m\{\theta(B)JA - \theta(A), JB) \quad (1.2)$$

It is simple to calculate directly that

$$\bar{\nabla}_A B = \nabla_A B + \theta(B)\{lA + mJA\}. \quad (1.3)$$

Description of Hopf hypersurfaces is discussed by magnitude of Hopf principal curvature. [16] [3] [5] [12] [14] [15].

2. Lightlike hypersurfaces

Gauss Weingarten formula for M and S(TM) are

$$\bar{\nabla}_A B = \nabla_A B + T(A,B)N. \tag{2.1}$$

$$\bar{\nabla}_A N = -Q_N A + \tau(A)N, \tag{2.2}$$

$$\nabla_A PB = \nabla^*_A PB + C(A,PB)\xi. \tag{2.3}$$

$$\nabla_A \xi = -Q^*_\xi A + \sigma(A)\xi. \tag{2.4}$$

For U, V, we have

$$U = -JN, \quad V = J\xi, \quad u(A) = g(A, V), \quad v(A) = g(A, U). \tag{2.5}$$

Operating J to vector field, A = SA + u(A)U

$$JA = FA + u(A)N, \tag{2.6}$$

Again, operating J to (2.6) and with (1.2), (1.3), (2.5), we find

$$F^2A = A - u(A)U \tag{2.7}$$

with (1.1), (1.2), (2.1), (2.7), we get

$$(\nabla_A g)(B, C) = T(A, B)\eta(C) + B(A, C)\eta(B) - l\{\theta(B)g(A, C) + \theta(C)g(A, B)\} - m\{\theta(B)\bar{g}(JA, C) + \theta(C)\bar{g}(JA, B)\}, \tag{2.8}$$

$$T(A, B) = l\{\theta(B)A + \theta(A)B\} + m\{\theta(B)FA - \theta(A)FB\}, \tag{2.9}$$

$$T(A, B) - T(B, A) = m\{\theta(B)u(A) - \theta(A)u(B)\} \tag{2.10}$$

where θ is 1-form.

Since T (A, B) = $\bar{g}(\bar{\nabla}_A B, \xi)$, therefore,

$$T(A,\xi) = 0, \tag{2.11}$$

Second fundamental forms are

$$T(A,B) = g(Q^*_\xi A, B) + mu(A)\theta(B), \quad \bar{g}(Q^*_\xi A, N) = 0, \tag{2.12}$$

$$C(A,PB) = g(Q_N A, PB) + \{l\eta(A) + mv(A)\}\theta PB, \quad \bar{g}(Q_N A, N) = 0 \tag{2.13}$$

Replacing A by ξ in (2.12), we get

$$Q^*_\xi \xi = 0,$$
$$\overline{\nabla}_A \xi = -Q^*_\xi A - \tau(A)\xi \tag{2.14}$$

Operating ∇_A to $F\xi = -V$ *and* $FV = \xi$, we have

$$(\nabla_A F)\xi = -\nabla_A V + F(Q^*_\xi A - \tau(A)V, \tag{2.15}$$
$$(\nabla_A F)V = -F\nabla_A V - Q^*_\xi A - \tau(A)\xi. \tag{2.16}$$

Applying $\overline{\nabla}_A$ to g (U, U) and g (V, V), we have

$$u(\nabla_A U) = 0, \; u(\nabla_A V) = 0. \tag{2.17}$$

3. Indefinite Cosympletic Manifold

Let M be an almost contact manifold equipped with an almost contact metric structure (ϕ, ξ, η, g) [19]

$$J^2 A = -A + \theta(A)\zeta, \; \overline{g}(JA, JB) = \overline{g}(A, B) - \theta(A), \theta(B), \theta(\zeta) = 1, \tag{3.1}$$

An almost contact metric structure is cosymplectic [14] if and only if both $\nabla\eta$ and $\nabla\phi$ vanish, where ∇ is the covariant differentiation with

respect to g.

$$d\eta = 0 \qquad d\phi = 0. \tag{3.2}$$

By (3.1) and (3.2), we have

$$\overline{\nabla}_A \zeta = A - \theta(A)\zeta, \; d\theta(A, B) = g(A, JB). \tag{3.3}$$

For two null vector field U, V,

$$U = -JN, V = J\xi. \tag{3.4}$$

Vector A = SA + u (A)U with,

$$u(A) = g(A, V); v(A) = g(A, U). \tag{3.5}$$

Operating J to (3.5),

$$JA = FA + u(A)N, \tag{3.6}$$

operating $\overline{\nabla}_A$ in (3.4), (3.6) with (2.1), (2.4), (3.4), (3.6), we derive

$$B(A, U) = C(A, V), \tag{3.7}$$

$$\nabla_A U = F\ (Q_N A) + \tau(A)U - v(A)\zeta, \tag{3.8}$$

$$\nabla_A V = F\ (Q^*_\xi A) - \tau(A)V - u(A)\zeta, \tag{3.9}$$

$$(\nabla_A F\ )B = u\ (\mathrm{B})Q_N A - T(A,\ B\ )U \tag{3.10}$$
$$+ \bar{g}(\mathrm{J}\,A,\ B)\ \zeta - \theta\ (B)\mathrm{F}\ \ A.$$

Operating $\bar{\nabla}_A$ on $g(\zeta,\xi) = 0$ $\bar{g}(\zeta, N) = 0$ and using (3.3), we obtain

$$T(A,\zeta) = 0, \qquad \mathrm{C}(A,\zeta) = \eta(A). \tag{3.11}$$

Theorem 1: A cosympletic manifold with an indefinite structure, a lightlike hypersurface M and its transversal connection with parallel F, is flat.

Proof: For parallel F, (3.10), yields

$$u(B)Q_N A - T(A,B)\,\mathrm{U} = 0. \tag{3.12}$$

Replacing A by U and B by V, we have $\lambda = 0.$ Therefore $\overline{M}$ is flat manifold.

Replacing B by U in (3.12), gives

$$Q_N \mathrm{A} = \sigma(A)U. \tag{3.13}$$

Scalar product with V to (3.12), yields

$$T(\ A,\ \ B\ ) = \mathrm{u}\ \ (\mathrm{B}\ )\ \ \sigma\ \ (\ A).$$

Equivalently $g(Q^*_\xi A,\ \ B) = \mathrm{g}(\sigma(A)\,\mathrm{V}, B).$

Since both $Q^*_\xi\ A$ and V belong to non-degenerate S(TM), hence we get

$$Q^*_\xi \mathrm{A} = \sigma(A)V. \tag{3.14}$$

With (3.13), (3.14), (2.10) gives $\lambda = \mu = 0.$ Hence

$$R\ (A,\ \ B)\ \ C = \{\sigma\ (B)\ \sigma\ (A) - \sigma\ (A)\,\sigma\ (B)\}u(C)\,\mathrm{U} = 0.$$

Giving M flat.

With (3.13), (3.8) FU = 0, gives $\nabla_A U = \tau(A)U$

With this and $\nabla_A \nabla_B U - \nabla_B \nabla_A U - \nabla_{[A,B]} U = 0,$ we have $d\tau = 0.$

4. Hopf lightlike hypersurfaces

A Hopf lightlike hypersurface is a lightlike hypersurface with smooth function f such that [16]

$$Q_{\xi}^{*}U = fU \tag{4.1}$$

(4.1) with scalar product with A and (3.5), gives,

$$\begin{aligned} T(A, U) &= \mathrm{f}\,\nu(A), \\ C(A,V) &= \mathrm{f}\,\nu(A), \\ \sigma(A) &= \mathrm{f}\,\nu(A). \end{aligned} \tag{4.2}$$

Lie recurrent F defined as [17]

$$(L_A \mathrm{F})\ \mathrm{B} = \nu(\mathrm{A})\ \mathrm{FB}. \tag{4.3}$$

Equivalently

$$(L_A \mathrm{F})\,\mathrm{B} = [\mathrm{A}, \mathrm{FB}] - \mathrm{F}[\mathrm{A}, \mathrm{B}] \tag{4.4}$$

If (L_A F) = 0, then F is Lie parallel.

Theorem 2: On Lie recurrent indefinite cosympletic manifold with ζ as tangent to M, F is Lie parallel.

Proof: Since $\overline{\nabla}$ is torsion free hence by (4.3) and (4.4), we get

$$\begin{aligned} (\nabla_A F)\ B &= \nabla_{FB} A - F\ \nabla_B A \\ &\quad + \sigma\ (A) FB. \end{aligned} \tag{4.5}$$

Equation (2.16), (4.5) with B = V, yields

$$\nabla_{\xi} A = -F(\nabla_A V - \nabla_V A) - Q_{\xi}^{*} A - \{\sigma(A) + \tau(A)\}\xi. \tag{4.6}$$

Again comparing (2.15) with (4.5), and replacing B by ξ, we get

$$u(\nabla_A V - \nabla_V A) = 0, \ \ \theta(\nabla_A V - \nabla_V A) = 0. \tag{4.7}$$

Applying F in (4.5) and with (2.7), (4.7), (4.6), we obtain $\sigma = 0$. Hence F is Lie parallel.

Definition: A cosympletic manifold is cosympletic space form if

$$\begin{aligned} \vec{R}(A, B)\ C &= \frac{(c-3)}{4}\{\bar{g}(B, C)\ A - \bar{g}(A, C)\ B\} \\ &\quad + \frac{(c+1)}{4}\{\bar{g}(A, JC)JB - \bar{g}(B, JC)JA \\ &\quad + 2\bar{g}(A, JB)JC + \theta(A)\,\theta(C)\ B - \theta(B)\,\theta(C)\ A \\ &\quad + \bar{g}(A, C)\,\theta(B)\zeta - \bar{g}(B, C)\,\theta(A)\zeta. \end{aligned} \tag{4.8}$$

With (1.2), (1.3) and (2.2), we obtain

$$\begin{aligned}\overline{R}(A, B)C = \overrightarrow{R}(A, B)C &+ (\nabla_A \theta)(C)\{l(B+mJB\} \\ &-(\nabla_B\theta)(C)\{l(A+mJA\}+\theta(C)\{(Al)B \\ &-(Bl)A+(Am)JB)-(Bm)JA) \\ &-\mathrm{m}\,[\theta(B)\mathrm{J}A-\theta(A)\mathrm{J}B]-2\mathrm{m}\overline{\mathrm{g}}(A,\mathrm{J}B)\zeta\}.\end{aligned} \tag{4.9}$$

Scalar product of this with ξ and N and with (3.2), (3.7), (4.1), (4.2), (4.8) we obtain

$$\begin{aligned}\nabla_A T(B, C)-\nabla_B T(A, C)&+\{\tau(A)-1\theta(A)\}T(B, C) \\ &-\{\tau(B)-1\theta(B)\}T(A, C) \\ &-m\{\theta(A)T(\mathrm{F}B, C)-\theta(B)T(\mathrm{F}A, C)\} \\ &-\mathrm{m}\{(\overline{\nabla}_A\theta)(C)u(B)-(\overline{\nabla}_B\theta)(C)u(A) \\ &-\theta(C)\{[A\mathrm{m}+\mathrm{m}\theta(\mathrm{X})]\mathrm{u}(B)-[Bm+m\theta(B)]\mathrm{u}(A)\} \\ &=\frac{(c+1)}{4}\{u(B)\overline{g}(A,\mathrm{J}C)-u(A)\overline{g}(B,\mathrm{J}C) \\ &+2\{u(C)\overline{\mathrm{g}}(A, JB)\}.\end{aligned} \tag{4.10}$$

Applying $\overline{\nabla}_A$ in $\theta(U)=0$ *and* $\theta(\xi)=0$ and using (2.14), we have

$$(\overline{\nabla}_A\theta)U=-\theta(\nabla_A U),\quad (\overline{\nabla}_A\theta)\xi=-\theta(Q^*_\xi A). \tag{4.11}$$

Theorem 3: For indefinite cosympletic manifold $\overline{M}$ with Hopf lightlike hypersurface M, $c=1$.

Proof: Taking A= U (4.6) and with (4.3), (4.4), we get

$$\nabla_\xi U=-Q^*_\xi U. \tag{4.12}$$

Taking scalar product with ζ , we get

$$\theta(\nabla_\xi U)=-\theta(Q^*_\xi U).$$

Comparing with (2.10), we have

$$\theta(\nabla_\xi U)=\theta(Q^*_\xi U)=0,\ T(U,\zeta)=m,\ T(\zeta,U)=0. \tag{4.13}$$

Scalar product of U to (4.12) with (2.17), yields

$$T(U,U)=0. \tag{4.14}$$

Operating ∇_ξ in (4.14) and using (2.10), (2.12), (4.12) with (4.13), we get

$$(\nabla_{\xi}T)(U,U) = 2g(Q_{\xi}^{*}U, Q_{\xi}^{*}U). \tag{4.15}$$

Applying ∇_U to $T(\xi,U)$ and with (2.4), (2.10) to (2.12), (4.13), we get

$$(\nabla_{U}T)(\xi,U) = g(Q_{\xi}^{*}U, Q_{\xi}^{*}U). \tag{4.16}$$

Replacing A by ξ, B and C by U in (4.10) and with (2.11), (4.11), (4.13), (4.15), (4.16), we get

$$g(Q_{\xi}^{*}U, Q_{\xi}^{*}U) = 3\left(\frac{c-1}{4}\right).$$

As M is Hopf lightlike hypersurface so $Q_{\xi}^{*}U = \lambda U$, hence $g(Q_{\xi}^{*}U, Q_{\xi}^{*}U) = 0$.

Therefore $\frac{c-1}{4} = 0$. Hence $c=1$.

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