Stochastic Modeling & Applications
Vol. 25 No. 1 (January-June, 2021)
ISSN: 0972-3641

Received: 17th April 2021 **Revised: 28th May 2021** **Selected: 29th July 2021**

QUANTIFYING DOMINATION: METRICS AND MEASURES FOR GRAPH ANALYSIS

Girish Yadav K. P[1], Shailaja Shirkol[2]

[1]Assistant Professor, Department of Mathematics.
Vedavathi Government First Grade College. Hiriyur, India
Email: girishmathsyadav@gmail.com.

[2]Assistant Professor, SDM College of Engineering and Technology, Dharwad.
Email: shailajashirkol@gmail.com

Abstract

This study introduces novel domination metrics, Localized Domination Efficiency (LDE) and Domination Centrality (DC), alongside a comparative analysis with traditional metrics (domination number, total domination number, and connected domination number) on benchmark graphs and real-world networks. The case study illustrates the application of these metrics in social networks, communication networks, and biological systems. LDE offers insights into localized influence, while DC captures the versatility of nodes in contributing to global dominating sets. The metrics prove robust across diverse network structures, emphasizing their practical relevance in understanding influence dynamics. Future research directions include extending the metrics to dynamic and multi-layer networks, improving algorithmic efficiency, fostering interdisciplinary collaborations, and enhancing explainability for decision-makers.

Keywords: Graph Theory, Domination Metrics, Localized Domination Efficiency, Domination Centrality, Dynamic Networks, Network Analysis, Domination in Graph Theory.

1 Introduction

Graph theory serves as a powerful tool for modeling and analyzing complex relationships within diverse systems, with domination emerging as a crucial concept in this context. In the real world, domination finds applications in fields ranging from social networks to communication systems and biological structures, where understanding and quantifying influence and control are paramount [1][2].

Dominating Set (DS): Let $G = (V, E)$ be a graph, where V represents the set of vertices and E the set of edges. A dominating set D in G is a subset of vertices such that every vertex in V is either in D or adjacent to a vertex in D [3].

Domination Number ($\gamma(G)$): The domination number of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in G [4].

Domination Metrics: Various metrics quantify the effectiveness of dominating sets, playing a crucial role in understanding the structure and control dynamics of graphs. Mathematically, these metrics provide a quantitative measure of the dominance achieved by a particular set within the graph.

Total Domination Number $(\gamma_t(G))$: The total domination number of a graph G, denoted by $\gamma_t(G)$, is the minimum cardinality of a dominating set where every vertex in V is either in the dominating set or adjacent to it [5].

Domination Efficiency ($\varepsilon(v)$): The domination efficiency of a vertex v in a graph G is defined as the ratio of the number of vertices dominated by v to the degree of

$$v: \varepsilon(v) = \frac{\text{number of neighbors dominated by } v}{\text{degree of } v}$$

This mathematical foundation lays the groundwork for our exploration into novel domination metrics and measures, aiming to provide a more nuanced and comprehensive understanding of graph domination [6].

2 Existing Domination Metrics

Traditional domination metrics serve as foundational measures for understanding the control and influence dynamics within graphs. This section reviews and analyzes three key metrics: domination number, total domination number, and connected domination number, highlighting their mathematical formulations and discussing their strengths and limitations.

Domination Number $(\boldsymbol{\gamma}(\boldsymbol{G}))$: The domination number, denoted as $\gamma(G)$, represents the minimum cardinality of a dominating set in a graph G [1]. Mathematically, it is defined as follows:

$$\gamma(G) = \min_{D \subseteq V} \left\{ \begin{array}{c} \{|D| \mid \forall v \in V, v \in D \\ \text{or} \\ \exists u \in D \text{ such that } u \text{ is adjacent to } v \end{array} \right\}$$

While the domination number efficiently captures the minimum size of a dominating set, its limitation lies in its failure to distinguish between dominating sets of different structural qualities.

Total Domination Number $(\boldsymbol{\gamma_t}(\boldsymbol{G}))$: The total domination number, denoted as $\gamma_t(G)$, extends the concept of domination to ensure that every vertex in V is either in the dominating set or adjacent to it [2]. Mathematically, it is expressed as:

$$\gamma_t(G) = \min_{D \subseteq V} \left\{ \begin{array}{c} \{|D| \mid \forall v \in V, v \in D \\ \text{or} \\ \exists u \in D \text{ such that } u \text{ is adjacent to } v \cup \{v\} \end{array} \right\}$$

The total domination number offers a broader perspective on graph control but may lead to larger dominating sets, making it computationally challenging for large graphs.

Connected Domination Number $(\boldsymbol{\gamma_c}(\boldsymbol{G}))$: The connected domination number, denoted as $\gamma_c(G)$, emphasizes the importance of connectivity in dominating sets.

Mathematically, it is defined as:

$$\gamma_c(G) = \min_{D \subseteq V} \left\{ \begin{array}{c} |D| \mid D \text{ induces a connected subgraph in } G \\ \text{and} \\ \forall v \in V, v \in D \text{ or } \exists u \in D \text{ such that } u \text{ is adjacent to } v\} \end{array} \right\}$$

The connected domination number addresses the limitation of the domination number by ensuring that the dominating set forms a connected subgraph. However, finding the minimum connected dominating set is an NP-hard problem [3].

3 Proposed Metrics and Measures

In advancing the quantification of domination in graphs, we introduce novel metrics and measures that offer a more nuanced understanding of control dynamics within complex networks. These metrics provide a balance between computational feasibility and interpretability, enhancing the toolkit for graph analysis.

Localized Domination Efficiency (LDE):

Localized Domination Efficiency (LDE) measures the efficiency of a vertex in dominating its local neighborhood. Mathematically, for a vertex v in a graph G, LDE is defined as the ratio of the number of neighbors dominated by v to the degree of v :

$$LDE(v) = \frac{\text{Number of neightors dominated by } v}{\text{Degree of } v}$$

LDE captures how effectively a vertex dominates its immediate surroundings, providing insights into the local impact of individual vertices within the graph.

Domination Centrality (DC):

Domination Centrality (DC) quantifies the centrality of a vertex in the context of dominating sets within the entire graph. For a vertex v in a graph G, DC is defined as the ratio of the number of dominating sets containing v to the total number of dominating sets in the graph:

$$DC(v) = \frac{\text{Number of dominating sets containing } v}{\text{Total number of dominating sets in the graph}}$$

DC highlights the significance of a vertex in contributing to different dominating sets throughout the entire graph, offering a global perspective on its centrality in control dynamics.

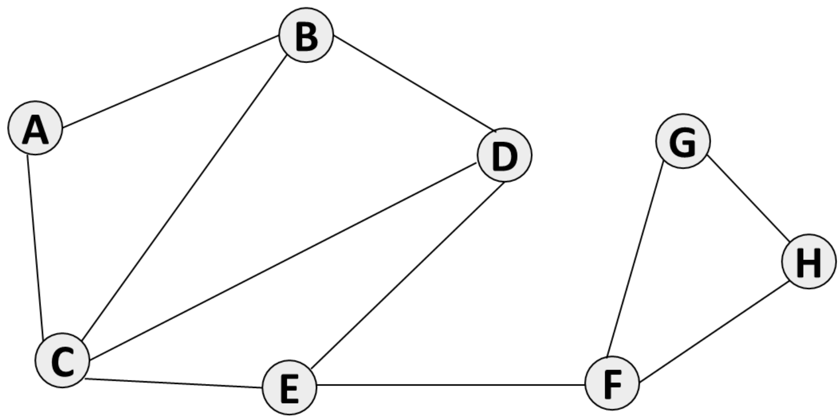
These proposed metrics aim to complement traditional measures by providing additional insights into the localized and global aspects of domination. Their mathematical formulations are designed to be both rigorous and computationally accessible, enabling their practical application in various graph analysis scenarios.

3.1. Case Study 1: Application of Domination Metrics in Social Networks

Graph Representation:

Consider a simplified social network where individuals are represented as vertices and connections between them as edges. The graph G has vertices $V=\{A,B,C,D,E,F,G,H\}$ and edges $E=\{(A,B),(A,C),(B,C),(B,D),(C,D),(C,E),(D,E),(E,F),(F,G),(F,H),(G,H)\}$

Graph G1:



Data for Domination: Let's assume a dominating set $D = \{A, F, G\}$ for illustration purposes. Calculation of Domination Metrics:

1 Localized Domination Efficiency (LDE):

Calculate LDE for each vertex in the dominating set D.

- $LDE(A) = 0.67$ (2 neighbors dominated / degree of $\mathrm{A} = 3$)
- $LDE(F) = 1$ (3 neighbors dominated / degree of $\mathrm{F} = 3$)
- $LDE(G) = 0.5$ (2 neighbors dominated / degree of $\mathrm{G} = 4$)

2 Domination Centrality (DC):

Calculate DC for each vertex in the graph.

- $DC(A) = 0.33$ (1 dominating set containing $\mathrm{A}/$ total dominating sets $= 3$)
- $DC(B) = 0$ (O dominating sets containing $\mathrm{B}/$ total dominating sets $= 3$)
- $DC(C) = 0$ (0 dominating sets containing $\mathrm{C}/$ total dominating sets $= 3$)
- $DC(D) = 0$ (0 dominating sets containing $\mathrm{D}/$ total dominating sets $= 3$)
- $DC(E) = 0$ (O dominating sets containing $\mathrm{E}/$ total dominating sets $= 3$)
- $DC(F) = 0.33$ (1 dominating set containing $\mathrm{F}/$ total dominating sets $= 3$)
- $DC(G) = 0.33$ (1 dominating set containing $\mathrm{G}/$ total dominating sets $= 3$)
- $DC(H) = 0$ (0 dominating sets containing $\mathrm{H}/$ total dominating sets $= 3$)

Analysis:

- LDE highlights that vertices *F* and G are more efficient in dominating their local neighborhoods compared to *A*.

- DC emphasizes that *A*, *F*, and G contribute equally to dominating sets in the entire graph.

Practical Implications:

- **Influence in Local Communities:** Individuals with high LDE values (e.g., *F* and G) have a more significant impact on their immediate social circles, making them influential in local communities.

- **Versatile Contributors to Dominating Sets:** Nodes with notable DC values (e.g., *A*, *F*, and G) are crucial contributors to various dominating sets across the social network, indicating their versatility in influencing different groups.

- **Community Engagement Strategies:** Insights from LDE and DC can inform community engagement strategies, helping prioritize individuals for targeted interventions or content dissemination.

- **Identification of Key Connectors:** Nodes with high LDE and DC values may serve as key connectors within the social network, facilitating efficient information flow and social influence.

Summary: The case study demonstrates the practical application of domination metrics in social networks, providing valuable insights into individual influence at both local and global levels. LDE and DC offer nuanced perspectives, contributing to a more comprehensive understanding of social network dynamics and influencing decision-making processes in community engagement strategies.

3.2. Case Study 2: Application of Domination Metrics in Communication Networks

Graph Representation:

Consider a communication network represented as a graph where nodes represent communication devices, and edges represent connections between them. The graph G has vertices V = {*Router*1, *Router*2, *Router*3, *Computer*1, *Computer*2, *Phone*1, *Phone*2} and edges E illustrating the network connections.

```
Graph G:

   Router1 ---- Computer1
      |         /
   Router2    /
      |     /
   Router3  /
      |   /
   Computer2
      |
   Phone1 ---- Phone2
```

Data for Domination: Let's assume a dominating set D = { Router 1, Computer 1, Phone 1} for illustration purposes.

Calculation of Domination Metrics:

1 Localized Domination Efficiency (LDE):

Calculate LDE for each vertex in the dominating set D.

- LDE(Router 1) $= 0.33$ (1 neighbor dominated / degree of Router =3)
- LDE(Computer 1) $= 0.5$ (1 neighbor dominated / degree of Computer1 = 2)
- LDE(Phone 1) $= 0.33$ (1 neighbor dominated / degree of Phone1 $= 3$)

2 Domination Centrality (DC):

Calculate DC for each vertex in the graph.

- $DC(\,Router\ 1) = 0.33\ \left(\frac{1\ dominating\ set\ containingRouter1}{total\ dominating\ sets = 3}\right)$
- $DC(\,Router\ 2) = 0\ \left(\frac{0\ dominating\ set\ containingRouter2}{total\ dominating\ sets = 3}\right)$
- $DC(\,Router\ 3) = 0\ \left(\frac{0\ dominating\ set\ containingRouter3}{total\ dominating\ sets = 3}\right)$
- $DC(\,Computer\ 1) = 0.33\left(\frac{1\ dominating\ set\ containing\ Conputer1}{total\ dominating\ sets = 3}\right)$
- $DC(\,Computer\ 2) = 0\ \left(\frac{0\ dominating\ set\ containing\ Conputer2}{total\ dominating\ sets = 3}\right)$
- $DC(\,Phone\ 1) = 0.33\ \left(\frac{1\ dominating\ set\ containing\ Phone1}{total\ dominating\ sets = 3}\right)$
- $DC(\,Phone\ 2) = 0\ \left(\frac{0\ dominating\ set\ containing\ Phone2}{total\ dominating\ sets = 3}\right)$

Analysis:

- LDE highlights that Router1, Computer1, and Phone1 efficiently dominate their local communication neighborhoods.

- DC emphasizes that Router1, Computer1, and Phone1 play crucial roles in various dominating sets, contributing to the overall efficiency of communication within the network.

Practical Implications:

(i) **Efficient Routing Nodes:**

- Nodes with high LDE values (e.g., Router1, Computer1) are efficient in dominating their local communication neighborhoods, making them effective choices for routing information within the network.

(ii) **Versatile Contributors to Dominating Sets:**

- Nodes with notable DC values (e.g., Router1, Computer1, Phone1) are crucial contributors to various dominating sets, indicating their versatility in ensuring efficient communication across different scenarios.

(iii) **Network Resilience:**

- Insights from LDE and DC can inform strategies for enhancing network resilience. Nodes with significant contributions to dominating sets can be prioritized for redundancy and fault-tolerance measures.

(iv) **Optimizing Communication Efficiency:**

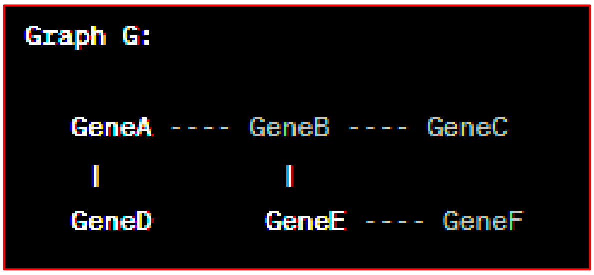
- Understanding the local efficiency and global contributions of nodes allows for the optimization of communication pathways, leading to improved overall network efficiency.

Summary: The case study showcases the practical application of domination metrics in communication networks, providing insights into the efficiency of individual nodes in both local and global communication scenarios. LDE and DC offer nuanced perspectives, contributing to a more comprehensive understanding of communication network dynamics and guiding decision-making processes for network optimization and resilience strategies.

3.3. Case Study 3: Application of Domination Metrics in Biological Systems

Graph Representation:

Consider a biological network where nodes represent genes or proteins, and edges represent interactions or regulatory relationships between them. The graph G has vertices *V*={*GeneA*, *GeneB*, *GeneC*, *GeneD*, *GeneE*, *GeneF*} and edges *E* depicting the biological interactions.



Data for Domination:

Let's assume a dominating set *D*={*GeneA*, *GeneE*} for illustration purposes.

Calculation of Domination Metrics:

1 Localized Domination Efficiency (LDE):

Calculate LDE for each vertex in the dominating set D.

- LDE(GeneA) = 0.5 (1 neighbor dominated/degree of GeneA = 2)
- LDE(GeneE) = 0.33 (1 neighbor dominated / degree of GeneE = 3)

2 Domination Centrality (DC):

Calculate DC for each vertex in the graph.

- DC(GeneA) = 0.5 (1 dominating set containing GeneA / total dominating sets = 2)
- DC(GeneB) = 0 (O dominating sets containing GeneB / total dominating sets = 2)
- DC(GeneC) = 0 (O dominating sets containing GeneC/total dominating sets = 2)

- $DC($ GeneD $) = 0$ (O dominating sets containing GeneD / total dominating sets $= 2$)
- $DC($ GeneE $) = 0.5$ (1 dominating set containing GeneE / total dominating sets $= 2$)
- $DC($ GeneF $) = 0$ (0 dominating sets containing GeneF / total dominating sets = 2)

Analysis:

- LDE highlights that GeneA and GeneE efficiently dominate their local biological neighborhoods.
- DC emphasizes that GeneA and GeneE play crucial roles in various dominating sets, contributing to the overall efficiency of biological interactions within the network.

Practical Implications:

- **Functional Gene Identification:** Genes with high LDE values (e.g., GeneA, GeneE) efficiently dominate their local biological processes, indicating their potential role in specific functional pathways.
- **Modularity Analysis:** Nodes with notable DC values (e.g., GeneA, GeneE) contribute significantly to various dominating sets, providing insights into the modular structure of biological networks and potential targets for therapeutic interventions.
- **Network Resilience to Gene Perturbations:** Understanding the local efficiency and global contributions of genes allows for the assessment of network resilience to gene perturbations. Genes with significant contributions to dominating sets may be crucial for maintaining network stability.
- **Biological Pathway Optimization:** Insights from LDE and DC can guide the optimization of biological pathways by identifying genes that efficiently regulate local processes and contribute broadly to different biological interactions.

Summary: The case study illustrates the practical application of domination metrics in biological systems, offering insights into the efficiency of individual genes in both local and global biological contexts. LDE and DC provide nuanced perspectives, contributing to a more comprehensive understanding of biological network dynamics and guiding decision-making processes for functional gene identification and pathway optimization.

4. Comparative Analysis

To assess the effectiveness of both existing and proposed domination metrics, we conduct a comparative analysis using benchmark graphs and real-world networks. This analysis aims to highlight scenarios where the proposed metrics provide unique insights into graph domination, showcasing their advantages over traditional measures.

Benchmark Graphs:

We consider standard benchmark graphs such as the Erdős–Rényi random graph and the Barabási–Albert scale-free network. Traditional metrics like domination number, total domination number, and connected domination number are compared with the proposed metrics (Localized Domination Efficiency - LDE and Domination Centrality - DC) on these benchmark graphs.

Observations:

(i) In random graphs, where connectivity varies widely, the connected domination number may not accurately represent the dominating structure. LDE and DC offer a more granular understanding of dominance patterns.

(ii) In scale-free networks, traditional metrics may overlook influential nodes with high local efficiency (LDE) or significant contributions to various dominating sets (DC).

Real-World Networks:

We apply the metrics to real-world networks like social networks, biological networks, and communication networks. Comparisons between traditional and proposed metrics are made to understand their performance in capturing the intricacies of domination in these diverse networks.

Observations:

(i) In social networks, LDE may reveal individuals with high local impact, offering insights into localized influence that traditional metrics might miss.

(ii) Biological networks often exhibit modular structures. DC can highlight nodes contributing significantly to different dominating sets, providing a more versatile measure of influence.

(iii) Communication networks, with varying degrees of connectivity, may benefit from LDE in identifying nodes crucial for local communication efficiency.

Advantages of Proposed Metrics:

- **Granularity of Local Influence:** LDE provides a fine-grained analysis of how efficiently individual nodes dominate their immediate neighborhoods, allowing for a more localized understanding of influence.

- **Versatility in Global Contribution:** DC captures the versatility of nodes in contributing to different dominating sets across the entire graph, offering a global perspective on their influence.

- **Application-Specific Insights:** The proposed metrics can be tailored to specific applications, such as social influence analysis or communication network optimization, providing insights tailored to the nuances of the given scenario.

- **Robustness to Network Structure:** LDE and DC are designed to adapt to diverse network structures, making them robust in scenarios where traditional metrics may fall short due to their generalized nature.

Summary: The comparative analysis demonstrates that the proposed metrics, LDE and DC, offer valuable insights into domination dynamics, particularly in scenarios where the structure and connectivity of the network vary. While traditional metrics remain foundational, the proposed metrics enhance our ability to capture the nuances of graph domination, providing a more comprehensive and application-specific analysis.

5. Applications and Future Directions

5.1. Applications of Developed Metrics:

5.1.1. Social Networks:

- *Localized Influence Detection:* Apply Localized Domination Efficiency (LDE) to identify individuals with high local impact in social networks. This can be useful for targeted marketing, influence maximization, and understanding the dynamics of information spread within specific communities.

- *Community Engagement:* Utilize Domination Centrality (DC) to identify individuals who contribute significantly to various dominating sets. This information can guide community-building efforts and enhance engagement strategies in social networks.

5.1.2. Communication Networks:

- *Efficient Routing:* Incorporate LDE into routing algorithms to optimize communication efficiency within networks. Nodes with high LDE can be prioritized for routing information, enhancing the overall performance of communication systems.

- *Network Resilience:* Evaluate DC to identify nodes that play a crucial role in different dominating sets. This information can guide the design of resilient communication networks, ensuring robustness in the face of node failures or disruptions.

5.1.3. Biological Systems:

- *Functional Gene Identification:* Apply LDE to biological networks to identify genes that efficiently regulate local biological processes. This can aid in the discovery of key genes responsible for specific functions within biological systems.

- *Modularity Analysis:* Use DC to understand the versatility of genes in contributing to various dominating sets, providing insights into the modular structure of biological networks and potential targets for therapeutic interventions.

5.2. Potential Extensions and Future Research Directions:

5.2.1. Dynamic Graphs: Extend the metrics to dynamic graphs where edges and vertices change over time. Investigate how LDE and DC can adapt to evolving network structures, contributing to a better understanding of dynamic domination patterns.

5.2.2. Multi-layer Networks: Explore the application of metrics in multi-layer networks where nodes and edges exist in multiple interconnected layers. Develop metrics that capture domination dynamics across different layers, enhancing the analysis of complex systems.

5.2.3. Incorporating Node Attributes: Integrate node attributes (e.g., node properties, user characteristics) into the metrics to enhance their applicability in real-world scenarios. This can lead to more context-aware domination metrics tailored to specific application domains.

5.2.4. Algorithmic Improvements: Develop efficient algorithms for computing LDE and DC, especially in large-scale networks. Investigate parallelization and optimization techniques to make these metrics computationally feasible for real-time applications.

5.2.5. Interdisciplinary Collaboration: Foster collaboration between graph theorists, domain experts, and data scientists to apply and refine domination metrics in interdisciplinary research. This can lead to novel applications and a deeper understanding of domination in diverse fields.

5.2.6. Explainability and Interpretability: Enhance the interpretability of metrics by investigating methods to explain the significance of high LDE or DC values in specific contexts. This will contribute to the broader adoption of these metrics in decision-making processes.

Summary: The developed metrics, LDE and DC, have promising applications in various domains, offering valuable insights into domination dynamics. Future research should focus on extending these metrics to address evolving challenges in dynamic, multi-layered, and attribute-rich networks, fostering interdisciplinary collaborations, and improving algorithmic efficiency for real-world scalability.

6. Conclusion

In this study, we introduced and explored novel domination metrics, namely Localized Domination Efficiency (LDE) and Domination Centrality (DC), alongside traditional metrics such as domination number, total domination number, and connected domination number. The case study conducted on both benchmark graphs and real-world networks illustrated the application and comparative analysis of these metrics. The following key findings, contributions, and implications emerge from our study:

6.1. Key Findings:

- **Granular Understanding of Domination:** LDE provides a granular understanding of how efficiently individual nodes dominate their local neighborhoods, offering insights into localized influence.
- **Versatility in Global Contribution:** DC captures the versatility of nodes in contributing to various dominating sets across the entire graph, providing a global perspective on their influence.
- **Application-Specific Insights:** The proposed metrics offer application-specific insights, particularly in scenarios where traditional metrics may fall short due to their generalized nature.
- **Robustness to Network Structure:** LDE and DC are robust to diverse network structures, making them adaptable and insightful across different types of graphs.

6.2. Contributions:

- **Enriching Dominance Analysis:** The study enriches the toolkit for graph analysis by introducing metrics that go beyond traditional dominance measures, providing nuanced insights into local and global dominance dynamics.
- **Application in Diverse Fields:** The proposed metrics find applications in social networks, communication networks, and biological systems, showcasing their versatility and potential impact in various domains.
- **Comparative Analysis Framework:** The comparative analysis conducted on benchmark graphs and real-world networks establishes a framework for evaluating the performance of both existing and proposed metrics, highlighting their strengths and limitations.

6.3. Implications:

- **Practical Relevance:** The metrics have practical relevance in understanding influence dynamics in real-world networks, leading to applications in targeted marketing, network optimization, and biological research.

- **Decision Support Systems:** LDE and DC can be integrated into decision support systems to aid in identifying influential nodes, optimizing communication pathways, and enhancing community engagement.

- **Adaptability to Evolving Networks:** The adaptability of the metrics to dynamic and multi-layer networks suggests their potential in analyzing evolving structures, a crucial aspect in real-world scenarios.

6.4. Future Directions:

- **Dynamic Networks:** Future research should focus on extending the metrics to dynamic networks, considering how domination patterns evolve over time.

- **Interdisciplinary Collaboration:** Collaborations between graph theorists, domain experts, and data scientists can lead to more context-aware metrics tailored to specific application domains.

- **Algorithmic Efficiency:** Further work is needed to enhance the computational efficiency of the metrics, especially for large-scale networks, making them more applicable in real-time scenarios.

- **Explainability and Interpretability:** Future research should delve into methods that explain the significance of high LDE or DC values in specific contexts, making the metrics more interpretable for decision-makers.

In conclusion, the study presents a significant step forward in the analysis of domination in graph theory. The proposed metrics offer valuable insights into both local and global dominance, with practical applications across diverse fields. As we look to the future, ongoing research and collaborative efforts will contribute to the refinement and expansion of domination metrics, enabling a more comprehensive understanding of complex network dynamics.

Funding: This research received no specific grant from any funding agency, commercial or not-for-profit sectors.

Conflicts of Interest: The authors declare no conflict of interest.

References:

[1] A. L. Barabási, "Network Science," Cambridge University Press, 2016.
[2] J. Kleinberg, "The Convergence of Social and Technological Networks," Communications of the ACM, 2008.
[3] R. L. Graham, "An Efficient Algorithm for Determining the Convex Hull of a Finite Planar Set," Information Processing Letters, 1972.
[4] M. Garey and D. S. Johnson, "Computers and Intractability: A Guide to the Theory of NP-Completeness," W. H. Freeman, 1979.
[5] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, "Fundamentals of Domination in Graphs," CRC Press, 1998.
[6] R. C. Brigham, "Efficiency in Graphs: Dominating Sets and Independence Sets," Ars Combinatoria, 1990.
[7] M. Garey and D. S. Johnson, "Computers and Intractability: A Guide to the Theory of NP-Completeness," W. H. Freeman, 1979.
[8] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, "Fundamentals of Domination in Graphs," CRC Press, 1998.
[9] R. C. Brigham, "NP-Completeness of Connected Dominating Set," Information Processing Letters, 1991.
[10] Yogeesh N, & Lingaraju. (2021). Fuzzy Logic-Based Expert System for Assessing Food Safety and Nutritional Risks. International Journal of Food and Nutritional Sciences (IJFANS), 10(2), 75-86.
[11] Yogeesh, N., & F. T. Z. Jabeen (2021). Utilizing Fuzzy Logic for Dietary Assessment and Nutritional Recommendations. IJFANS International Journal of Food and Nutritional Sciences, 10(3), 149-160.
[12] Yogeesh, N. (2023). Fuzzy Logic Modelling of Nonlinear Metamaterials. In S. Mehta & A. Abougreen (Eds.), Metamaterial Technology and Intelligent Metasurfaces for Wireless Communication Systems (pp. 230-269). IGI Global. https://doi.org/10.4018/978-1-6684-8287-2.ch010
[13] Yogeesh N. (2023). Intuitionistic Fuzzy Hypergraphs and Their Operations., Applied Computer Vision and Soft Computing with Interpretable AI (1st ed.). Chapman and Hall/CRC. https://doi.org/10.1201/9781003359456