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Predicting the T20 World Cup 2021: Advanced Insights Using Fuzzy TOPSIS and Fuzzy SAW Methods

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Abstract: This article presents a prediction for the outcome of the Twenty20 Cricket World Cup 2021. The prediction is a significant study topic due to its complexity and reliance on unpredictable factors such as weather and pitch conditions. Due to the numerous and sometimes overlooked aspects that impact the final outcome of a cricket match, accurately predicting the precise and partially truth-based results of such contests is a difficult task. Hence, this study expects a decision-making approach that considers multiple criteria. These strategies were mainly used to address the consequences of the Twenty20 Cricket World Cup 2021. The modified fuzzy Technique for Order of Preference by Similarity to Ideal Solution(TOPSIS) approach, incorporating the triangular number methodology, and the modified fuzzy Simple Additive Weighting (SAW) method were employed to forecast the true ranking. Based on the proposed approach, India is considered the frontrunner to emerge victorious in the Twenty20 World Cup in 2021. Team Bangladesh has the lowest probability of winning.

Keywords: T20 World Cup, Cricket, Multi Criteria Decision Making, Prediction, TOPSIS, Fuzzy SAW Method.

Introduction

The sport ofcricket has a known history beginning in the late 16th century. Having originated in south-east England, it became the country's national sport in the 18th century and has developed globally in the 19th and 20th centuries [1]. International matches have been played since 1844 andTest cricketbegan, retrospectively recognized, in 1877[2]. Cricket is the world's second most popular spectator sport afterassociation football(soccer). Governance is by theInternational Cricket Council(ICC) which has over one hundred countries and territories in membership although only twelve currently playTest cricket [3-5].

Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" (1 or 0). Boolean logic on which the modern computer is based. Fuzzy Logic is basicallya multi-valued logic that allows intermediate values to be defined between conventional evaluations like yes/no, true/false, black/white, etc.[6-7].

A fuzzy subset of a universe X (a fuzzy set) is a mathematical object A described by its (generalized) characteristic function (membership function)

$$\mu_A: X \rightarrow [0, 1]$$

The classical membership degrees are represented by 1 (is a member) and 0 (not a member) Alternative notation: F(X) denotes the set of all fuzzy subsets of a universe X.

Fuzzy SAW Method

SAW methodcan help in decision making for a certain case, and the calculation that generates the greatest value will be chosen as the best alternative[9]. Other than SAW, also by using anothermethodwas the CPImethodwhich is one of the calculatingmethodsin decision making based on performance index[8]. The various steps of Fuzzy SAW method are presented as follows.

STEP-1: Choosing the criteria that will be used in decision-making, (C_j; j = 1, 2…m) and then choosing committee of experts for decision-making, (E_k; k = 1, 2 … n).

STEP -2: Assigned the suitable rating of each criterion by the experts in terms of linguistic variable.

STEP-3: Determine the fuzzy decision matrix $DM_{IJ}$for all criteria in terms of fuzzy triangular numbers.

$$DM_{IJ} = \begin{bmatrix} X_{11} & \cdots & X_{1n} \\ \vdots & \ddots & \vdots \\ X_{m1} & \cdots & X_{mn} \end{bmatrix}$$

STEP-4: Determine the average fuzzy scores (A_{jk}), (A_{jk}) = $(f_{j1}^k + f_{j2}^k + \cdots f_{jn}^k)$ / n; j = 1, 2…m; k = 1, 2…n. De-fuzzified values (e), e = (a + b + c) / 3

And normalized weight (W_j) of each criterion.

$$W_j = \frac{\text{defuzzified values}}{\text{sum of total defuzzified values}}$$

$$W_j = \frac{e_j}{\sum_{j=1}^{n} e_j}; \qquad j = 1,2 \ldots n.$$

STEP-5: Assigned the suitable rating in terms of linguistic variables by the experts for each maintenance strategy (A_i; i = 1, 2…) of all the criteria.

STEP-6: Determine average fuzzy score and de-fuzzified scores of each strategy of all the criteria.

STEP-7: Determine decision matrix for all criteria and maintenance strategy [X_{ij}].

STEP-8: Determine normalized matrix for all criteria and maintenance strategy [R_{ij}] .

$$r_{ij} = {x_{ij}}/{\max(x_{1j}, x_{2j}, x_{3j})} \qquad i = 1,2.3 \ldots$$

STEP-9: Determine the Total Scores (TS) for eachmaintenance strategy by Simple Additive Weighting (SAW) method. $TS = [R_{ij}]\,[W_j]$

STEP-10: The final results obtained from the ranking the sum of normalized matrix [R_{ij}] multiplication with thenormalized weight (W_j) in order toobtain the greatest value is selected as the best maintenance strategy (A_i) as a solution.

STEP-11: Final scores and ranks for selection of maintenance strategy problem.

Modified Fuzzy Saw Method

Modified technique is used in Step 4

Determine the average fuzzy scores (A_{jk}), (A_{jk}) = $(a_{j1}^k + a_{j2}^k + a_{j3}^k)$ / n; j = 1, 2…m; k = 1, 2…n, $a_{1ij} = \min\{a_{1ij}\}$, $a_{2ij} = \sum_{k=1}^{p} \frac{a_{2ijk}}{p}$, $a_{3ij} = \max\{a_{3ij}\}$.

De-fuzzified values (e), e = (a + b + c) / 3

And normalized weight (W_j) of each criterion.

$$W_j \quad = \frac{\text{defuzzified values}}{\text{sum of total defuzzified values}} = \frac{e_j}{\sum_{\mathrm{j=1}}^{\mathrm{n}} e_j} \;; \qquad j = 1,2 \ldots n.$$

Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

TheTechnique for Order of Preference by Similarity to Ideal Solution(TOPSIS) is amulti-criteria decision analysismethod, which was originally developed by Ching-Lai Hwang and Yoon in 1981[10-11]with further developments by Yoon in 1987,and Hwang, Lai and Liu in 1993[12]. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution (PIS)and the longest geometric distance from the negative ideal solution (NIS)[13]. The TOPSIS system is useful for leaders to structure the issues to be tackled, direct examinations, correlations and positioning of the choices. The established TOPSIS strategy tackles issues in which all choice information are known and spoken to by fresh numbers. Most genuine issues, in any case, have a more confounded structure. In view of the first TOPSIS strategy, numerous different expansions have been proposed, offering help for interim or fuzzy criteria, interim or fuzzy weights to demonstrated imprecision, vulnerability, absence of data or dubiousness [14-15, 20].

STEP 1: First, performance ratings and weights are evaluated with linguistic terms. These linguistic ratings, employed by experts to represent the performances under certain criteria, are very low (VL), low (L), medium low (ML), medium (M), medium high (MH), high (H) and very high (VH).Choosing committee of experts for decision-making.(E_k; k = 1, 2 … n) and thento alternative M_i against choosing the criteria (C_j; j = 1, 2…m) where $G_{ijk} = (g_{1ijk}, g_{2ijk}, g_{3ijk})$ is a triangular fuzzy number.

STEP 2: Then $G_{ij}$is the average performance rating of alternative A_i against criterion c_j using by the extension principle.

$G_{ijk} = (g_{1ijk}, g_{2ijk}, g_{3igk}) = {}^{1}/_{p} \times (G_{ij1} + G_{ij2} + \cdots + G_{ijp})$

By the extension principle, we have

$g_{1ij} = \sum_{k=1}^{p} \frac{g_{1ijk}}{p}, \quad g_{2ij} = \sum_{k=1}^{p} \frac{g_{2ijk}}{p}, \ g_{3ij} = \sum_{k=1}^{p} \frac{g_{3ijk}}{p}$

STEP 3: A decision-making matrix G is composed of the performance ratings of alternative $A_1, A_2, A_3, A_4, A_5, A_6$; that is, $G = [G_{ij}]_{mXn}$. $[G_{i1}, G_{i2}, .. G_{in}]$denotes the performance ratings of alternative M_i on all criteria. Let M^- and M^+ denote the negative ideal solution and ideal solution respectively.

Thus$M^- = [G_1^-, G_2^-, \ldots G_n^-]$and $M^+ = [G_1^+, G_2^+, \ldots G_n^+]$

Where, $G_j^- = Lo[G_{i1}, G_{i2}, .. G_{im}]$ and$G_j^+ = Up[G_{i1}, G_{i2}, .. G_{im}]$ for j = 1, 2, . . ., n.

STEP 4: we compute the distance from alternatives to the ideal solution (or negative ideal solution). Let d_{ij}^- and d_{ij}^+ be the distance from $G_{ij}$to $G_j^-$and G_j^+respectively; where i = 1, 2, . . .,m; j = 1, 2, . . ., n. By the definition Let ($A = (a_1, a_2, a_3)$ and ($B = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. A distance measure function $d(A, B)$

$$d(A, B) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

STEP 5: Let $W_{jk} = (w_{1jk}, w_{2jk}, w_{3jk})$ denote the weight evaluated by expert E_k under criterion C_j, where j = 1, 2, . . .,n; k = 1, 2, . . . , p. Assume W_j to be the average weight on criterion C_j ; thus

$W_j = (w_{1j}, w_{2j}, w_{3j}) = {}^{1}/_{p} \times (W_{j1} + W_{j2} + \cdots + W_{jp})$ where j = 1, 2, . . ., n.

By the extension principle, we have

$w_{1j} = \sum_{k=1}^{p} \frac{w_{1jk}}{p}, \ w_{2j} = \sum_{k=1}^{p} \frac{w_{2jk}}{p}, w_{3j} = \sum_{k=1}^{p} \frac{w_{3jk}}{p}$

STEP 6: D_i^- and D_i^+ express the weighted distance from alternative M_i to negative ideal solution M^- and ideal solution M^+ respectively.

$D_i^- = \sum_{j=1}^{n} W_j \times d_{ij}^-$ and $D_i^+ = \sum_{j=1}^{n} W_j \times d_{ij}^+$, where i = 1, 2, . . ., m.

STEP 7: Thus, the weighted distance of M_i can be expressed by $[D_i^-, D_i^+]$. Let

$LD^- = Lo(\{D_1^-, D_2^-, \dots D_m^-\})$ and $UD^- = Up(\{D_1^-, D_2^-, \dots D_m^-\})$

$LD^+ = Lo(\{D_1^+, D_2^+, \dots D_m^+\})$ and $UD^+ = Up(\{D_1^+, D_2^+, \dots D_m^+\})$

STEP 8: By the two operations of Lo and Up, we know that the negative ideal solution is $[LD^-, UD^+]$ and the ideal solution is $[UD^-, LD^+]$ for weighted distance values of all alternatives.

STEP 9: Let M_i^- denote the distance from $[D_i^-, D_i^+]$ to $[LD^-, UD^+]$, and M_i^+ denote the distance from $[D_i^-, D_i^+]$ to $[UD^-, LD^+]$.

Define $M_i^- = d(D_i^-, LD^-) + d(D_i^+, UD^+)$ and $M_i^+ = d(D_i^-, UD^-) + d(D_i^+, LD^+)$, where i = 1, 2, . . ., m.

STEP 10: the closeness coefficient A_i^* of alternative A_i is defined:

$A_i^* = \frac{A_i^-}{A_i^- + A_i^+}$ where i = 1, 2, . . ., m. If $A_i^* = 0$, alternative A_i would be the negative ideal solution. In contrast, $A_i^* = 1$ denotes A_i to be ideal solution.

Modified Fuzzy TOPSIS

The step 2 is modified in Fuzzy TOPSIS method

STEP 2: Then G_{ij} is the performance rating of alternative A_i against criterion c_j using by the;

$$G_{ijk} = (g_{1ijk}, g_{2ijk}, g_{3igk})$$

We have

$g_{1ij} = \min\{g_{1ij}\}$, $g_{2ij} = \sum_{k=1}^{p} \frac{g_{2ijk}}{p}$, $g_{3ij} = \max\{g_{3ij}\}$

Case Study: Winner of Twenty20 Cricket World Cup 2021

The 7th version of the World Cup is nearest to the conclusion and some of the best players of Cricket teams have graced the stage in India with 16 teams taking part. Data is collected up to 20th July 2020, after the WC 2016 of the following teams[16-19]: A_1 = Australia, A_2 = England, A_3 = India, A_4 = Pakistan, A_5 = South Africa, A_6 = New Zealand, A_7 = Sri Lanka, A_8 = Bangladesh and A_9 = West Indies.

D_1, D_2 *and* D_3 are Decision makers and Following Criteria are considered in the calculations which plays important role in each game: C_1 = T20 Ranking, C_2 = Overall winning % in T20, C_3 = Winning % in T20world cup, C_4 = Top 20 best man, C_5 = Top 20 bowler, C_6 = World cup win and C_7 = Top 20 all-rounder. Linguistic variables used for significant weight of every criterion are as followsVery low (VL) is (0.0,0.0,0.1), Low (L) is (0.0,0.1,0.3), Medium low (ML) is (0.1,0.3,0.5), Medium (M) is (0.3,0.5,0.7), Medium High (MH) is (0.5,0.7,0.9), High (H) is (0.7,0.9,1.0) and Very high (VH) is (0.9,1.0,1.0).

Solution of Numerical Example by using Fuzzy SAW Method

Table 1

Choosing the criteria that will be used in decision-making, (C_j; j = 1, 2...m) and then choosing committee of experts for decision-making, (A_k; k = 1, 2 ... n).

C_j	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
C_1	(VH,VH,H)	(VH,H,H)	(H,H,H)	(H,H,MH)	(H,MH,MH)	(MH,MH,MH)	(MH,MH,M)	(MH,M,ML)	(M,ML,L)

C_2	(MH,ML,M)	(MH,ML,ML)	(H,H,MH)	(H,MH,MH)	(MH,MH,M)	(ML,ML,M)	(ML,H,M)	(VL,L,VL)	(M,L,VL)
C_3	(MH,M,ML)	(ML,M,M)	(VH,H,MH)	(MH,M,M)	(H,H,M)	(M,M,MH)	(VH,H,H)	(M,L,VL)	(MH,MH,M
C_4	(VH,VH,H)	(VH,H,H)	(VH,H,H)	(H,H,MH)	(H,H,H)	(VH,H,H)	(ML,L,VL)	(ML,L,VL)	(H,H,MH)
C_5	(VH,VH,H)	(H,H,H)	(H,H,H)	(H,H,H)	(H,H,H)	(VH,H,H)	(ML,L,VL)	(ML,L,VL)	(H,H,MH)
C_6	(VH,H,H)	(VH,H,H)	(VH,H,H)	(VH,H,H)	(H,H,H)	(H,H,H)	(VH,H,H)	(H,MH,VL)	(VH,VH,H)
C_7	(H,H,H)	(VH,VH,H)	(VH,H,H)	(VH,H,H)	(VH,H,H)	(VH,VH,H)	(ML,L,VL)	(H,H,H)	(H,H,H)

Table 2

Suitable rating of each criterion given by experts in the forms of linguistic variables and Fuzzy decision matrix determine for all criteria in forms of fuzzy triangular numbers

C_j	Linguistic Variables			Fuzzy Triangular Numbers		
	D_1	D_2	D_3	D_1	D_2	D_3
C_1	MH	H	L	(0.5,0.7 0.9)	(0.7,0.9,1.0)	(0.0,0.1,0.3)
C_2	VH	ML	VL	(0.9,1.0,1.0)	(0.1,0.3,0.5)	(0.0,0.0,0.1)
C_3	L	M	MH	(0.0,0.1,0.3)	(0.3,0.5,0.7)	(0.5,0.7,0.9)
C_4	VH	VH	VH	(0.9,1.0,1.0)	(0.9,1.0,1.0)	(0.9,1.0,1.0)
C_5	L	ML	ML	(0.0,0.0,0.1)	(0.1,0.3,0.5)	(0.1,0.3,0.5)
C_6	MH	ML	M	(0.5,0.7,0.9)	(0.1,0.3,0.5)	(0.3,0.5,0.7)
C_7	MH	MH	L	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.0,0.1,0.3)

Table 3

Determine the average fuzzy scores (A_{jk})*, de-fuzzified values*(e) *and normalized weight*(W_k) *of each criterion*

C_j	A_{jk}			e	W_k
C_1	0.400	0.567	0.733	0.567	0.156
C_2	0.333	0.433	0.533	0.433	0.119
C_3	0.267	0.433	0.633	0.444	0.122
C_4	0.900	1.000	1.000	0.967	0.266
C_5	0.067	0.200	0.367	0.211	0.058
C_6	0.300	0.500	0.700	0.500	0.137
C_7	0.333	0.500	0.700	0.511	0.140
Sum =				3.633	

Table 4

Suitable ratings are given in form of linguistic variables for each criterion by the experts anddetermine average fuzzy score and de-fuzzified scores of each criterion

Criteria	Strategies	Decision Makers			Average Fuzzy scores			De-fuzzifiedscores
		D_1	D_2	D_3	D_1	D_2	D_3	
C_1	A_1	VH	VH	H	0.833	0.967	1.000	0.933
	A_2	VH	H	H	0.767	0.933	1.000	0.900
	A_3	H	H	H	0.700	0.900	1.000	0.867
	A_4	H	H	MH	0.700	0.900	1.000	0.867
	A_5	H	MH	MH	0.567	0.767	0.933	0.755
	A_6	MH	MH	MH	0.500	0.700	0.900	0.700
	A_7	MH	MH	M	0.433	0.633	0.833	0.633
	A_8	MH	M	ML	0.300	0.500	0.700	0.500

	A_9	M	ML	L	0.133	0.300	0.500	0.311
C_2	A_1	MH	ML	M	0.300	0.500	0.700	0.500
	A_2	MH	ML	ML	0.233	0.433	0.633	0.433
	A_3	H	H	MH	0.633	0.833	0.967	0.811
	A_4	H	MH	MH	0.567	0.767	0.933	0.755
	A_5	MH	MH	M	0.433	0.633	0.833	0.633
	A_6	ML	ML	M	0.167	0.367	0.567	0.367
	A_7	ML	H	M	0.367	0.567	0.733	0.556
	A_8	VL	L	VL	0.000	0.033	0.167	0.067
	A_9	M	L	VL	0.100	0.200	0.367	0.223
C_3	A_1	MH	M	ML	0.300	0.500	0.700	0.500
	A_2	ML	M	M	0.233	0.433	0.633	0.433
	A_3	VH	H	MH	0.700	0.867	0.967	0.844
	A_4	MH	M	M	0.367	0.567	0.767	0.567
	A_5	H	H	M	0.567	0.767	0.900	0.744
	A_6	M	M	MH	0.367	0.567	0.767	0.567
	A_7	VH	H	H	0.767	0.933	1.000	0.900
	A_8	M	L	VL	0.100	0.200	0.367	0.222
	A_9	MH	MH	M	0.433	0.633	0.833	0.633
C_4	A_1	VH	VH	H	0.833	0.967	1.000	0.933
	A_2	VH	H	H	0.767	0.933	1.000	0.900
	A_3	VH	H	H	0.767	0.933	1.000	0.900
	A_4	H	H	MH	0.633	0.833	0.967	0.811
	A_5	H	H	H	0.700	0.900	1.000	0.867
	A_6	VH	H	H	0.767	0.933	1.000	0.900
	A_7	ML	L	VL	0.033	0.133	0.300	0.155
	A_8	ML	L	VL	0.033	0.133	0.300	0.155
	A_9	H	H	MH	0.633	0.833	0.967	0.811
C_5	A_1	VH	VH	H	0.833	0.967	1.000	0.933
	A_2	H	H	H	0.700	0.900	1.000	0.867
	A_3	H	H	H	0.700	0.900	1.000	0.867
	A_4	H	H	H	0.700	0.900	1.000	0.867
	A_5	H	H	H	0.700	0.900	1.000	0.867
	A_6	VH	H	H	0.767	0.933	1.000	0.900
	A_7	ML	L	VL	0.033	0.133	0.300	0.155
	A_8	ML	L	VL	0.033	0.133	0.300	0.155
	A_9	H	H	MH	0.633	0.833	0.967	0.811
C_6	A_1	VH	H	H	0.767	0.933	1.000	0.900
	A_2	VH	H	H	0.767	0.933	1.000	0.900
	A_3	VH	H	H	0.767	0.933	1.000	0.900
	A_4	VH	H	H	0.767	0.933	1.000	0.900
	A_5	H	H	H	0.700	0.900	1.000	0.867
	A_6	H	H	H	0.700	0.900	1.000	0.867
	A_7	VH	H	H	0.767	0.933	1.000	0.900
	A_8	H	MH	VL	0.400	0.533	0.667	0.533

	A_9	VH	VH	H	0.833	0.967	1.000	0.933
C_7	A_1	H	H	H	0.700	0.900	1.000	0.867
	A_2	VH	VH	H	0.833	0.967	1.000	0.933
	A_3	VH	H	H	0.767	0.933	1.000	0.900
	A_4	VH	H	H	0.767	0.933	1.000	0.900
	A_5	VH	H	H	0.767	0.933	1.000	0.900
	A_6	VH	VH	H	0.833	0.967	1.000	0.933
	A_7	ML	L	VL	0.167	0.267	0.433	0.289
	A_8	H	H	H	0.700	0.900	1.000	0.867
	A_9	H	H	H	0.700	0.900	1.000	0.867

Table 5

Decision matrix determine and maintenance strategy $[X_{ij}]$for all criteria.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.933	0.500	0.500	0.933	0.933	0.900	0.867
A_2	0.900	0.433	0.433	0.900	0.867	0.900	0.933
A_3	0.867	0.811	0.844	0.900	0.867	0.900	0.900
A_4	0.867	0.755	0.567	0.811	0.867	0.900	0.900
A_5	0.755	0.633	0.744	0.867	0.867	0.867	0.900
A_6	0.700	0.367	0.567	0.900	0.900	0.867	0.933
A_7	0.633	0.556	0.900	0.155	0.155	0.900	0.289
A_8	0.633	0.556	0.222	0.155	0.155	0.533	0.867
A_9	0.311	0.223	0.633	0.811	0.811	0.933	0.867

Table 6

Normalize matrix determines and maintenance strategy $[R_{ij}]$ for all criteria.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	1.000	0.616	0.556	1.000	1.000	0.964	0.929
A_2	0.964	0.533	0.481	0.964	0.929	0.964	1.000
A_3	0.929	1.000	0.938	0.964	0.929	0.964	0.964
A_4	0.929	0.930	0.631	0.869	0.929	0.964	0.964
A_5	0.809	0.780	0.827	0.929	0.929	0.929	0.964
A_6	0.750	0.452	0.630	0.964	0.964	0.929	1.000
A_7	0.678	0.685	1.000	0.166	0.166	0.964	0.309
A_8	0.535	0.082	0.246	0.166	0.166	0.571	0.929
A_9	0.333	0.274	0.678	0.869	0.869	1.000	0.929

By simple Additive Weighting (SAW) method calculate the Total Scores (TS) for every maintenance strategy. $TS = [R_{ij}][W_j]$

$$\begin{bmatrix} 1.000 & 0.616 & 0.556 & 1.000 & 1.000 & 0.964 & 0.929 \\ 0.964 & 0.533 & 0.481 & 0.964 & 0.929 & 0.964 & 1.000 \\ 0.929 & 1.000 & 0.938 & 0.964 & 0.929 & 0.964 & 0.964 \\ 0.929 & 0.930 & 0.631 & 0.869 & 0.929 & 0.964 & 0.964 \\ 0.809 & 0.780 & 0.827 & 0.929 & 0.929 & 0.929 & 0.964 \\ 0.750 & 0.452 & 0.630 & 0.964 & 0.929 & 0.929 & 1.000 \\ 0.678 & 0.685 & 1.000 & 0.166 & 0.166 & 0.964 & 0.309 \\ 0.535 & 0.082 & 0.246 & 0.166 & 0.166 & 0.571 & 0.929 \\ 0.333 & 0.274 & 0.678 & 0.869 & 0.869 & 1.000 & 0.929 \end{bmatrix} \begin{bmatrix} 0.156 \\ 0.119 \\ 0.122 \\ 0.266 \\ 0.058 \\ 0.137 \\ 0.140 \end{bmatrix}$$

Table 7

For selection of problems find final scores and ranks.

Strategy	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
Final Score	0.88312	0.854	0.955	0.884	0.8831	0.827	0.538	0.385	0.715
Ranking	3	5	1	2	4	6	8	9	7

$$A_3 > A_4 > A_1 > A_5 > A_2 > A_6 > A_8 > A_9 > A_7$$

Its shows that A_3 gives better results as compare to others.

Numerical Example Solved by Modified Fuzzy SAW Method

The above example is solved by modified fuzzy saw method, the first three step are same.

Table 8

Determine the fuzzy scores (A_{jk}) *, defuzzified values(e) and normalized weight* (W_j)*of each criterion*$r_{ij} = (a_{ij},\ b_{ij}, c_{ij}) a_{ij} = min\{a_{ij}\}, b_{ij} = 1/k \sum\{b_{ij}\},\ c_{ij} = max\{c_{ij}\}$

Criteria (C_j)	Fuzzy scores(A_{jk})			De-fuzzified value (e)	Normalized Weight (W_j)
C_1	0.0	0.567	1.0	0.522	0.144
C_2	0.0	0.433	1.0	0.477	0.132
C_3	0.0	0.433	0.9	0.444	0.122
C_4	0.9	1.000	1.0	0.967	0.267
C_5	0.0	0.2	0.5	0.233	0.064
C_6	0.1	0.5	0.9	0.5	0.138
C_7	0.0	0.5	0.9	0.467	0.129

Sum=3.61

Step 5 are same as previous example.

Table 9

$r_{ij} = (a_{ij},\ b_{ij},\ \ c_{ij}) a_{ij} = min\{a_{ij}\},\ b_{ij} = 1/k \sum\{b_{ij}\} c_{ij} = max\{c_{ij}\}$

Criteria	Strategies	Average Fuzzy scores			De-fuzzifiedscores
		D_1	D_2	D_3	
	A_1	0.700	0.967	1.000	0.889
	A_2	0.700	0.933	1.000	0.878
	A_3	0.700	0.900	1.000	0.867
C_1	A_4	0.500	0.833	1.000	0.778
	A_5	0.500	0.767	0.900	0.722
	A_6	0.500	0.700	0.900	0.700
	A_7	0.300	0.633	0.900	0.611

	A_8	0.100	0.500	0.900	0.500
	A_9	0.000	0.300	0.700	0.333
C_2	A_1	0.100	0.500	0.900	0.500
	A_2	0.100	0.433	0.900	0.477
	A_3	0.500	0.833	1.000	0.778
	A_4	0.500	0.767	1.000	0.757
	A_5	0.100	0.633	0.700	0.477
	A_6	0.100	0.367	0.700	0.389
	A_7	0.100	0.567	1.000	0.556
	A_8	0.000	0.033	0.300	0.111
	A_9	0.000	0.200	0.700	0.300
C_3	A_1	0.100	0.500	0.900	0.500
	A_2	0.100	0.433	0.700	0.411
	A_3	0.500	0.867	1.000	0.786
	A_4	0.300	0.567	0.900	0.589
	A_5	0.300	0.767	0.900	0.656
	A_6	0.300	0.567	0.900	0.589
	A_7	0.700	0.933	1.000	0.878
	A_8	0.000	0.200	0.700	0.300
	A_9	0.300	0.633	0.900	0.611
C_4	A_1	0.700	0.967	1.000	0.889
	A_2	0.700	0.933	1.000	0.878
	A_3	0.700	0.933	1.000	0.878
	A_4	0.500	0.833	1.000	0.778
	A_5	0.700	0.900	1.000	0.867
	A_6	0.700	0.933	1.000	0.878
	A_7	0.000	0.133	0.500	0.211
	A_8	0.000	0.133	0.500	0.211
	A_9	0.500	0.833	1.000	0.778
C_5	A_1	0.700	0.967	1.000	0.889
	A_2	0.700	0.900	1.000	0.867
	A_3	0.700	0.900	1.000	0.867
	A_4	0.500	0.900	1.000	0.800
	A_5	0.700	0.900	1.000	0.867
	A_6	0.700	0.933	1.000	0.878
	A_7	0.000	0.133	0.500	0.211
	A_8	0.000	0.133	0.500	0.211
	A_9	0.500	0.833	1.000	0.778
C_6	A_1	0.700	0.933	1.000	0.878
	A_2	0.700	0.933	1.000	0.878
	A_3	0.700	0.933	1.000	0.878
	A_4	0.700	0.933	1.000	0.878
	A_5	0.700	0.900	1.000	0.867
	A_6	0.700	0.900	1.000	0.867
	A_7	0.700	0.933	1.000	0.878

	A_8	0.000	0.533	1.000	0.511
	A_9	0.700	0.967	1.000	0.889
C_7	A_1	0.700	0.900	1.000	0.867
	A_2	0.700	0.967	1.000	0.889
	A_3	0.700	0.933	1.000	0.878
	A_4	0.700	0.933	1.000	0.878
	A_5	0.700	0.933	1.000	0.878
	A_6	0.700	0.967	1.000	0.889
	A_7	0.000	0.267	0.900	0.389
	A_8	0.700	0.900	1.000	0.867
	A_9	0.700	0.900	1.000	0.867

Table 10

Decision matrix and maintenance strategy $[X_{ij}]$*determine for all criteria.*

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.889	0.500	0.500	0.889	0.889	0.878	0.867
A_2	0.878	0.477	0.411	0.878	0.867	0.878	0.889
A_3	0.867	0.778	0.786	0.878	0.867	0.878	0.878
A_4	0.778	0.757	0.589	0.778	0.800	0.878	0.878
A_5	0.722	0.477	0.656	0.867	0.867	0.867	0.878
A_6	0.700	0.389	0.589	0.878	0.878	0.867	0.889
A_7	0.611	0.556	0.878	0.211	0.211	0.878	0.389
A_8	0.500	0.111	0.300	0.211	0.211	0.511	0.867
A_9	0.333	0.300	0.611	0.778	0.778	0.889	0.867
Sum	6.278	4.345	5.320	6.368	6.368	7.524	7.402

Table 11

Normalized matrix and maintenance strategy $[R_{ij}]$ *determine for all criteria.*

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	1.000	0.643	0.569	1.000	1.000	0.988	0.975
A_2	0.988	0.613	0.468	0.988	0.975	0.988	1.000
A_3	0.975	1.000	0.895	0.988	0.975	0.988	0.988
A_4	0.875	0.973	0.671	0.875	0.900	0.988	0.988
A_5	0.812	0.613	0.747	0.975	0.975	0.975	0.988
A_6	0.787	0.500	0.671	0.988	0.988	0.975	1.000
A_7	0.687	0.843	1.000	0.237	0.237	0.988	0.438
A_8	0.562	0.143	0.342	0.237	0.237	0.575	0.975
A_9	0.375	0.386	0.696	0.875	0.875	1.000	0.975

By simple Additive Weighting (SAW) method calculate the Total Scores (TS) for every maintenance strategy, $TS = [R_{ij}][W_j]$

$$
\begin{bmatrix}
1.000 & 0.643 & 0.569 & 1.000 & 1.000 & 0.988 & 0.975 \\
0.988 & 0.613 & 0.468 & 0.988 & 0.975 & 0.988 & 1.000 \\
0.975 & 1.000 & 0.895 & 0.988 & 0.975 & 0.988 & 0.988 \\
0.875 & 0.973 & 0.671 & 0.875 & 0.900 & 0.988 & 0.988 \\
0.812 & 0.613 & 0.747 & 0.975 & 0.975 & 0.975 & 0.988 \\
0.787 & 0.500 & 0.671 & 0.988 & 0.988 & 0.975 & 1.000 \\
0.687 & 0.843 & 1.000 & 0.237 & 0.237 & 0.988 & 0.438 \\
0.562 & 0.143 & 0.342 & 0.237 & 0.237 & 0.575 & 0.975 \\
0.375 & 0.386 & 0.696 & 0.875 & 0.875 & 1.000 & 0.975
\end{bmatrix}
\begin{bmatrix}
0.144 \\ 0.132 \\ 0.122 \\ 0.267 \\ 0.064 \\ 0.138 \\ 0.129
\end{bmatrix}
$$

Table 12

For selection of problems find final scores and ranks.

Strategy	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
Final Scores	0.89141	0.87182	0.97158	0.89132	0.87371	0.85177	0.6035	0.4251	0.74326
Ranks	2	5	1	3	4	6	8	9	7

$$A_3 > A_1 > A_4 > A_5 > A_2 > A_6 > A_9 > A_7 > A_8$$

So, the best performance is A_3.

Numerical Example solved by Fuzzy TOPSIS Method

Table 13

Aggregated Alternative and criteria weightage Fuzzy decision matrix, which is already taken from the previous example.Suitable rating assigned in the form of linguistic variables for each criterion by the experts

Criteria	Strategies	Average Fuzzy scores		
		D_1	D_2	D_3
	A_1	0.833	0.967	1.000
	A_2	0.767	0.933	1.000
	A_3	0.700	0.900	1.000
	A_4	0.700	0.900	1.000
C_1	A_5	0.567	0.767	0.933
	A_6	0.500	0.700	0.900
	A_7	0.433	0.633	0.833
	A_8	0.300	0.500	0.700
	A_9	0.133	0.300	0.500
	A_1	0.300	0.500	0.700
	A_2	0.233	0.433	0.633
	A_3	0.633	0.833	0.967
	A_4	0.567	0.767	0.933
C_2	A_5	0.433	0.633	0.833
	A_6	0.167	0.367	0.567
	A_7	0.367	0.567	0.733
	A_8	0.000	0.033	0.167
	A_9	0.100	0.200	0.367
	A_1	0.300	0.500	0.700
	A_2	0.233	0.433	0.633
C_3	A_3	0.700	0.867	0.967
	A_4	0.367	0.567	0.767
	A_5	0.567	0.767	0.900

	A_6	0.367	0.567	0.767
	A_7	0.767	0.933	1.000
	A_8	0.100	0.200	0.367
	A_9	0.433	0.633	0.833
C_4	A_1	0.833	0.967	1.000
	A_2	0.767	0.933	1.000
	A_3	0.767	0.933	1.000
	A_4	0.633	0.833	0.967
	A_5	0.700	0.900	1.000
	A_6	0.767	0.933	1.000
	A_7	0.033	0.133	0.300
	A_8	0.033	0.133	0.300
	A_9	0.633	0.833	0.967
C_5	A_1	0.833	0.967	1.000
	A_2	0.700	0.900	1.000
	A_3	0.700	0.900	1.000
	A_4	0.700	0.900	1.000
	A_5	0.700	0.900	1.000
	A_6	0.767	0.933	1.000
	A_7	0.033	0.133	0.300
	A_8	0.033	0.133	0.300
	A_9	0.633	0.833	0.967
C_6	A_1	0.767	0.933	1.000
	A_2	0.767	0.933	1.000
	A_3	0.767	0.933	1.000
	A_4	0.767	0.933	1.000
	A_5	0.700	0.900	1.000
	A_6	0.700	0.900	1.000
	A_7	0.767	0.933	1.000
	A_8	0.400	0.533	0.667
	A_9	0.833	0.967	1.000
C_7	A_1	0.700	0.900	1.000
	A_2	0.833	0.967	1.000
	A_3	0.767	0.933	1.000
	A_4	0.767	0.933	1.000
	A_5	0.767	0.933	1.000
	A_6	0.833	0.967	1.000
	A_7	0.167	0.267	0.433
	A_8	0.700	0.900	1.000
	A_9	0.700	0.900	1.000

Now we taken the positive and negative ideal solution

$G_1^+ = (0.833,0.967,1.000) G_1^- = (0.133,0.300,0.500)$

$G_2^+ = (0.633,0.833,0.967) G_2^- = (0.00\ ,0.033,0.167)$

$G_3^+ = (0.767,0.933,1.000) G_3^- = (0.100,0.200,0.367)$

$G_4^+ = (0.833,0.967,1.000) \quad G_4^- = (0.033,0.133,0.300)$

$G_5^+ = (0.833,0.967,1.000) G_5^- = (0.033,0.133,0.300)$
$G_6^+ = (0.833,0.967,1.000) G_6^- = (0.400,0.533,0.667)$
$G_7^+ = (0.833,0.967,1.000)$ $G_7^- = (0.033,0.133,0.300)$

we find the distance between two measures by using distance formula.

$$d(A,B) = \sqrt{\left(\frac{1}{3}\right)[(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^3]}$$

Table 14

Distance between two measures

	A_1		A_2		A_3	
	$d(G_{1j},G_j^+)$	$d(G_{1j},G_j^-)$	$d(G_{2j},G_j^+)$	$d(G_{2j},G_j^-)$	$d(G_{3j},G_j^+)$	$d(G_{3j},G_j^-)$
C_1	0.000	0.628	0.043	0.592	0.086	0.557
C_2	0.312	0.444	0.379	0.379	0	0.748
C_3	0.406	0.283	0.473	0.218	0.105	0.655
C_4	0	0.783	0.043	.746	0.043	0.746
C_5	0	0.780	0.086	0.713	0.086	0.713
C_6	0.043	0.368	0.043	0.368	0.043	0.368
C_7	0.086	0.598	0	0.780	0.043	0.746
	A_4		A_5		A_6	
	$d(G_{4j},G_j^+)$	$d(G_{4j},G_j^+)$	$d(G_{5j},G_j^+)$	$d(G_{5j},G_j^+)$	$d(G_{6j},G_j^+)$	$d(G_{6j},G_j^+)$
C_1	0.140	0.140	0.196	0.443	0.196	0.445
C_2	0.057	0.057	0.181	0.575	0.445	0.316
C_3	0.341	0.341	0.161	0.549	0.341	0.349
C_4	0.140	0.140	0.086	0.713	0.043	0.746
C_5	0.086	0.086	0.086	0.713	0.043	0.746
C_6	0.043	0.043	0.083	0.334	0.086	0.334
C_7	0.043	0.043	0.043	0.746	0	0.780
	A_7		A_8		A_9	
	$d(G_{7j},G_j^+)$	$d(G_{7j},G_j^+)$	$d(G_{8j},G_j^+)$	$d(G_{8j},G_j^+)$	$d(G_{9j},G_j^+)$	$d(G_{9j},G_j^+)$
C_1	0.316	0.322	0.444	0.190	0.628	0
C_2	0.346	0.497	0.748	0	0.590	0.161
C_3	0	0.679	0.679	0	0.277	0.415
C_4	0.780	0	0.780	0.039	0.140	0.657
C_5	0.780	0	0.780	0	0.140	0.657
C_6	0.046	0.368	0.403	0	0	0.403
C_7	0.780	0	0.086	0.713	0.086	0.713

Table 15

The average weights by against seven criteriagiven by the expertise

	$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{D_3}$	Weights
C_1	MH	H	L	$W_1 = (0.400,0.567,0.733)$
C_2	VH	M	VL	$W_2 = (0.333,0.433,0.533)$
C_3	L	M	H	$W_3 = (0.267,0.433,0.633)$
C_4	VH	VH	VH	$W_4 = (0.900,1.000,1.000)$
C_5	L	ML	ML	$W_5 = (0.067,0.200,0.700)$

C_6	MH	MH	M	$W_6 = (0.300,0.500,0.700)$
C_7	MH	MH	L	$W_7 = (0.333,0.500,0.700)$

NextFind the weight distance value by using formula

$D_i^+ = \sum_{i=1}^{m} W_j \times d_{ij}^+$ and $D_i^- = \sum_{i=1}^{m} W_j \times d_{ij}^-$

$D_1^+ = (0.253836,0.3266578,0.513594)$ $D_1^- = (1.541107,2.092867,2.881315)$

$D_2^+ = (0.32706,0.474997,0.666235)$ $D_2^- = (1.510524,2.056765,2.822637)$

$D_3^+ = (0.134116,0.197427,0.292903)$ $D_3^- = (1.714802,2.368918,3.24648)$

$D_4^+ = (0.325009,0.451914,0.609254)$ $D_4^- = (1.514802,2.069152,2.813252)$

$D_5^+ = (0.304941,0.426918,0.578554)$ $D_5^- = (1.553347,2.133473,2.946811)$

$D_6^+ = (0.385013,0.54607,0.730006)$ $D_6^- = (1.457733,1.99246,2.76353)$

$D_7^+ = (1.269418,1.67799,2.309818)$ $D_7^- = (0.585994,0.875782,1.177708)$

$D_8^+ = (1.511775,2.050139,2.82224)$ $D_8^- = (0.348529,0.50323,0.67737)$

$D_9^+ = (0.685647,0.942487,1.248335)$ $D_9^- = (0.328436,0.460408,0.646708)$

Thus, the weighted distance of $M_i$can be expressed by

$UD^+ = (1.511775,2.050139,2.822243)$ $UD^- = (1.724758,2.368918,3.24648)$

$LD^+ = (0.134116,0.197427,0.292903)$ $LD^- = (0.328436,0.460408,0.646708)$

Next, we calculate the distance by using distance formula

$$d(A,B) = \sqrt{\left(\frac{1}{3}\right)[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^3]}$$

$d(D_1^+, UD^+) = 1.8149984$ $d(D_1^+, LD^+) = 0.1630316$

$d(D_2^+, UD^+) = 1.6865143$ $d(D_2^+, LD^+) = 0.2907744$

$d(D_3^+, UD^+) = 1.9772073$ $d(D_3^+, LD^+) =0$

$d(D_4^+, UD^+) = 1.7185320$ $d(D_4^+, LD^+) = 0.2590249$

$d(D_5^+, UD^+) = 1.7440790$ $d(D_5^+, LD^+) = 0.2334122$

$d(D_6^+, UD^+) = 1.6237068$ $d(D_6^+, LD^+) = 0.3538174$

$d(D_7^+, UD^+) = 0.3914976$ $d(D_7^+, LD^+) = 1.5862874$

$d(D_8^+, UD^+) = 0$ $d(D_8^+, LD^+) =1.9772073$

$d(D_9^+, UD^+) = 1.2092100$ $d(D_9^+, LD^+) = 0.7685811$

$d(D_1^-, UD^-) = 0.2847673$ $d(D_1^-, LD^-) = 1.744160$

$d(D_2^-, UD^-) = 0.3281151$ $d(D_2^-, LD^-) = 1.7010148$

$d(D_3^-, UD^-) = 0$ $d(D_3^-, LD^-) = 2.0290342$

$d(D_4^-, UD^-) = 0.3274281$ $d(D_4^-, LD^-) = 1.7019042$

$d(D_5^-, UD^-) = 0.2412589$ $d(D_5^-, LD^-) = 1.7879249$

$d(D_6^-, UD^-) = 0.3856871$ $d(D_6^-, LD^-) = 1.6435155$

$d(D_7^-, UD^-) = 1.6130774$ $d(D_7^-, LD^-) = 0.4166664$

$d(D_8^-, UD^-) = 1.9979259$ $d(D_8^-, LD^-) = 0.032545$

$d(D_9^-, UD^-) = 2.0290342$ $d(D_9^-, LD^-) = 0$

The distance values M_i^+ and M_i^-calculated as

$A_1^+ = d(D_1^+, LD^+) + d(D_1^-, UD^-)$ =0.4477989 $A_1^- = d(D_1^-, LD^-) +$
$d(D_1^+, UD^+)$ =3.559158

$A_2^+ = d(D_2^+, LD^+) + d(D_2^-, UD^-) = 0.6188895$ $A_2^- = d(D_2^-, LD^-) +$
(D_2^+, UD^+) =3.3875291

$A_3^+ = d(D_3^+, LD^+) + d(D_3^-, UD^-)$ =0.0000 $A_3^- = d(D_3^-, LD^-) +$
$d(D_3^+, UD^+)$ =4.006241
$A_4^+ = d(D_4^+, LD^+) + d(D_4^-, UD^-) = 0.586453$ $A_4^- = d(D_4^-, LD^-) + d(D_4^+, UD^+) =$
3.4204362
$A_5^+ = d(D_5^+, LD^+) + d(D_5^-, UD^-)$ =0.4746711 $A_5^- = d(D_5^-, LD^-) + d(D_5^+, UD^+) =$
3.5320039
$A_6^+ = d(D_6^+, LD^+) + d(D_6^-, UD^-) = 0.7395045$ $A_6^- = d(D_6^-, LD^-) + d(D_6^+, UD^+) =$
3.2672223
$A_7^+ = d(D_7^+, LD^+) + d(D_7^-, UD^-)$ =3.1993648 $A_7^- = d(D_7^-, LD^-) + d(D_7^+, UD^+) =$
0.808164
$A_8^+ = d(D_8^+, LD^+) + d(D_8^-, UD^-)$ =3.9751332 $A_8^- = d(D_8^-, LD^-) +$
$d(D_8^+, UD^+)$ =0.0325454
$A_9^+ = d(D_9^+, LD^+) + d(D_9^-, UD^-)$ =2.7976153 $A_9^- = d(D_9^-, LD^-) + d(D_9^+, UD^+) =$
1.20921

The closeness coefficient A_i^* is find as

$$A_i^* = \frac{A_i^-}{A_i^- + A_i^+}$$

$A_1^* = \frac{A_1^-}{A_1^- + A_1^+}$ =0.888244,$A_2^* = \frac{A_2^-}{A_2^- + A_2^+}$ =0.845525,$A_3^* = \frac{A_3^-}{A_3^- + A_3^+}$ =1,

$A_4^* = \frac{A_4^-}{A_4^- + A_4^+}$ =0.853638,$A_5^* = \frac{A_5^-}{A_5^- + A_5^+}$ =0.875957,$A_6^* = \frac{A_6^-}{A_6^- + A_6^+}$ =0.815434,

$A_7^* = \frac{A_7^-}{A_7^- + A_7^+}$ =0.201661,$A_8^* = \frac{A_8^-}{A_8^- + A_8^+}$ =0.008120, $A_9^* = \frac{A_9^-}{A_9^- + A_9^+}$ =0.301787

Table 16

For selection of problems find final scores and ranks.

Strategy	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
Final Scores	0.8882	0.8455	1	0.8536	0.8759	0.8154	0.2016	0.008	0.3017
Ranks	2	5	1	4	3	6	8	9	7

$$A_3 > A_1 > A_5 > A_4 > A_2 > A_6 > A_9 > A_7 > A_8$$

So,A_3 is best.

Numerical Example solved by modified Fuzzy TOPSIS

Table 17

$a_{ij} = min\{a_{ij}\}$,$b_{ij} = 1/k \sum\{b_{ij}\}$, $c_{ij} = max\{c_{ij}\}$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.7,0.967,1.0)	(0.7,0.933,1.0)	(0.7,0.9,1.0)	(0.5,0.833,1.0)	(0.5,0.767,1.0)
C_2	(0.1,0.5,0.9)	(0.1,0.433,0.9)	(0.5,0.833,1.0)	(0.5,0.767,1.0)	(0.3,0.633.0.9)
C_3	(0.1,0.5,0.9)	(0.1,0.433,0.7)	(0.5,0.867,1.0)	(0.3,0.567,0.9)	(0.3,0.767,1.0)
C_4	(0.7,0.967,1.0)	(0.7,0.933,1.0)	(0.7,0.933,1.0)	(0.5,0.833,1.0)	(0.7,0.9,1.0)
C_5	(0.7,0.967,1.0)	(0.7,0.9,1.0)	(0.7,0.9,1.0)	(0.7,0.9,1.0)	(0.7,0.90,1.0)
C_6	(0.7,0.933,1.0)	(0.7,0.933,1.0)	(0.7,0.933,1.0)	(0.7,0.933,1.0)	(0.7,0.9,1.0)
C_7	(0.7,0.9,1.0)	(0.7,0.967,1.0)	(0.7,0.933,1.0)	(0.7,0.933,1.0)	(0.7,0.933,1.0)

	A_6	A_7	A_8	A_9
C_1	(0.5,0.7,0.9)	(0.3,0.633,0.9)	(0.1,0.5,0.9)	(0.0,0.3,0.7)
C_2	(0.1,0.367,0.7)	(0.1,0.567,1.0)	(0.0,0.033,0.3)	(0.0,0.2,0.7)

C_3	(0.3,0.567,0.9)	(0.7,0.933,1.0)	(0.0,0.2,0.7)	(0.3,0.633,0.9)
C_4	(0.7,0.933,1.0)	(0.0,0.133,0.5)	(0.0,0.133,0.5)	(0.5,0.833,1.0)
C_5	(0.7,0.933,1.0)	(0.0,0.133,0.5)	(0.0,0.133,0.5)	(0.5,0.833,1.0)
C_6	(0.7,0.9,1.0)	(0.7,0.933,1.0)	(0.0,0.533,1.0)	(0.7,0.967,1.0)
C_7	(0.7,0.967,1.0)	(0.0,0.267,0.9)	(0.7,0.9,1.0)	(0.7,0.90,1.0)

$G_1^+ = (0.7,0.967,1.000)$ $G_1^- = (0.0,0.3,0.7)$
$G_2^+ = (0.5,0.833,1.0)$ $G_2^- = (0.0,0.033,0.3)$
$G_3^+ = (0.7,0.933,1.000)$ $G_3^- = (0.0,0.20,0.7)$
$G_4^+ = (0.7,0.967,1.000)$ $G_4^- = (0.0,0.133,0.5)$
$G_5^+ = (0.7,0.967,1.000)$ $G_5^- = (0.0,0.533,1.0)$
$G_6^+ = (0.7,0.967,1.000)$ $G_6^- = (0.0,0.267,0.9)$
$G_7^+ = (0.7,0.967,1.000)$ $G_7^- = (0.033,0.133,0.300)$

By using following formula

$$d(A,B) = \sqrt{\left(\frac{1}{3}\right)[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^3]}$$

Table 18

Distance between two measures

	A_1		A_2		A_3	
	$d(G_{1j}, G_j^+)$	$d(G_{1j}, G_j^-)$	$d(G_{2j}, G_j^+)$	$d(G_{2j}, G_j^-)$	$d(G_{3j}, G_j^+)$	$d(G_{3j}, G_j^-)$
C_1	0.000	0.456	0.019	0.571	0.038	0.559
C_2	0.305	0.652	0.331	0.420	0	0.678
C_3	0.431	0.297	0.483	0.146	0.121	0.511
C_4	0	0.602	0.019	0.678	0.019	0.640
C_5	0	0.665	0.038	0.665	0.038	0.665
C_6	0.019	0.465	0.019	0.465	0.019	0.465
C_7	0.038	0.560	0	0.574	0.019	0.560
	A_4		A_5		A_6	
	$d(G_{4j}, G_j^+)$	$d(G_{4j}, G_j^+)$	$d(G_{5j}, G_j^+)$	$d(G_{5j}, G_j^+)$	$d(G_{6j}, G_j^+)$	$d(G_{6j}, G_j^+)$
C_1	0.138	0.140	0.163	0.548	0.201	0.387
C_2	0.038	0.057	0.173	0.341	0.394	0.306
C_3	0.318	0.341	0.250	0.404	0.318	0.297
C_4	0.138	0.140	0.038	0.442	0.081	0.496
C_5	0.038	0.086	0.019	0.678	0.019	0.678
C_6	0.019	0.043	0.083	0.456	0.038	0.456
C_7	0.019	0.043	0.019	0.560	0	0.574
	A_7		A_8		A_9	
	$d(G_{7j}, G_j^+)$	$d(G_{7j}, G_j^+)$	$d(G_{8j}, G_j^+)$	$d(G_{8j}, G_j^+)$	$d(G_{9j}, G_j^+)$	$d(G_{9j}, G_j^+)$
C_1	0.306	0.283	0.442	0.173	0.584	0
C_2	0.277	0.511	0.678	0	0.496	0.250
C_3	0	0.610	0.610	0	0.294	0.325
C_4	0.691	0	0.691	0	0.138	0.574
C_5	0.691	0	0.691	0	0.138	0.574

C_6	0.019	0.465	0.475	0	0	0.475
C_7	0.574	0	0.038	0.547	0.192	0.459

Next the average weights by against seven criteria.
$W_1 = (0.0,0.567,1.0)$
$W_2 = (0.0,0.5,1.0)$
$W_3 = (0.0,0.5,1.0)$
$W_4 = (\ 0.900,1.000,1.000)$
$W_5 = (0.0,0.233,0.500)$
$W_6 = (0.300,0.633,0.900)$
$W_7 = (0.0,0.500,0.900)$

Find the weight distance value by using formula
$D_i^+ = \sum_{i=1}^{m} W_j \times d_{ij}^+$ and $D_i^- = \sum_{i=1}^{m} W_j \times d_{ij}^-$

$D_1^+ = (0.005,0.399,0.787)$ $D_1^- = (0.761,2.079,3.189)$
$D_2^+ = (0.022,0.457,0.888)$ $D_2^- = (0.749,2.021,3.083)$
$D_3^+ = (0.022,0.131,0.231)$ $D_3^- = (0.715,2.280,3.643)$
$D_4^+ = (0.129,0.424,0.685)$ $D_4^- = (0.681,2.064,3.262)$
$D_5^+ = (0.045,0.379,0.684)$ $D_5^- = (0.534,1.851,2.988)$
$D_6^+ = (0.444,0.979,1.437)$ $D_6^- = (0.583,1.750,2.752)$
$D_7^+ = (0.627,1.463,2.153)$ $D_7^- = (0.139,1.015,1.822)$
$D_8^+ = (0.764,2.066,3.228)$ $D_8^- = (0.0,0.371,0.665)$
$D_9^+ = (0.124,0.992,1.753)$ $D_9^- = (0.659,1.525,2.276)$

Thus, the weighted distance from alternatives
$UD^+ = (0.764,2.066,3.228)$ $UD^- = (0.715,2.280,3.643)$
$LD^+ = (0.022,0.131,0.231)$ $LD^- = (0.000,0.371,0.665)$
By using distance formula, $d(A,B) = \sqrt{\left(\frac{1}{3}\right)[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^3]}$
$d(D_1^+, UD^+) = 0.552, d(D_1^+, LD^+) = 0.0.356$
$d(D_2^+, UD^+) = 0.516, d(D_2^+, LD^+) = 0.423$
$d(D_3^+, UD^+) = 0.657, d(D_3^+, LD^+) = 0.000$
$d(D_4^+, UD^+) = 0.430, d(D_4^+, LD^+) = 0.318$
$d(D_5^+, UD^+) = 0.522, d(D_5^+, LD^+) = 0.298$
$d(D_6^+, UD^+) = 1.223, d(D_6^+, LD^+) = 0.885$
$d(D_7^+, UD^+) = 0.716, d(D_7^+, LD^+) = 1.394$
$d(D_8^+, UD^+) = 0.000, (D_8^+, LD^+) = 2.104$
$d(D_9^+, UD^+) = 1.116, d(D_9^+, LD^+) = 1.011$
$d(D_1^-, UD^-) = 0.287, d(D_1^-, LD^-) = 1.813$
$d(D_2^-, UD^-) = 0.356, d(D_2^-, LD^-) = 1.744$
$d(D_3^-, UD^-) = 0.000, d(D_3^-, LD^-) = 2.083$
$d(D_4^-, UD^-) = 0.253, d(D_4^-, LD^-) = 1.832$
$d(D_5^-, UD^-) = 0.463, d(D_5^-, LD^-) = 1.619$
$d(D_6^-, UD^-) = 0.603, d(D_6^-, LD^-) = 1.482$
$d(D_7^-, UD^-) = 1.3226, d(D_7^-, LD^-) = 0.768$

$d(D_8^-, UD^-) = 2.0835, d(D_8^-, LD^-) = 0.000$
$d(D_9^-, UD^-) = 0.902, d(D_9^-, LD^-) = 1.205$

From the previous distance values A_i^+ and A_i^- calculated

$A_1^+ = d(D_1^+, LD^+) + d(D_1^-, UD^-)$ =0.643 $A_1^- = d(D_1^-, LD^-) + d(D_1^+, UD^+)$ =2.365
$A_2^+ = d(D_2^+, LD^+) + d(D_2^-, UD^-) = 0.779$, $A_2^- = d(D_2^-, LD^-) + d(D_2^+, UD^+)$ =2.26
$A_3^+ = d(D_3^+, LD^+) + d(D_3^-, UD^-)$ =0.000 $A_3^- = d(D_3^-, LD^-) + d(D_3^+, UD^+)$ =2.74
$A_4^+ = d(D_4^+, LD^+) + d(D_4^-, UD^-) = 0.571$ $A_4^- = d(D_4^-, LD^-) + d(D_4^+, UD^+) = 2.262$
$A_5^+ = d(D_5^+, LD^+) + d(D_5^-, UD^-)$ =0.761 $A_5^- = d(D_5^-, LD^-) + d(D_5^+, UD^+) = 2.141$
$A_6^+ = d(D_6^+, LD^+) + d(D_6^-, UD^-)$ =1.488 $A_6^- = d(D_6^-, LD^-) + d(D_6^+, UD^+)$ =2.705
$A_7^+ = d(D_7^+, LD^+) + d(D_7^-, UD^-)$ =2.7166 $A_7^- = d(D_7^-, LD^-) + d(D_7^+, UD^+) = 1.484$
$A_8^+ = d(D_8^+, LD^+) + d(D_8^-, UD^-)$ =4.1875 $A_8^- = d(D_8^-, LD^-) + d(D_8^+, UD^+)$ =0.000
$A_9^+ = d(D_9^+, LD^+) + d(D_9^-, UD^-)$ =1.913 $A_9^- = d(D_9^-, LD^-) + d(D_9^+, UD^+) = 2.321$

The closeness coefficient $A_i^* = \frac{A_i^-}{A_i^- + A_i^+}$,

$A_1^* = \frac{A_1^-}{A_1^- + A_1^+}$ =0.786, $A_2^* = \frac{A_2^-}{A_2^- + A_2^+}$ =0.743, $A_3^* = \frac{A_3^-}{A_3^- + A_3^+}$ =1
$A_4^* = \frac{A_4^-}{A_4^- + A_4^+}$ =0.571, $A_5^* = \frac{A_5^-}{A_5^- + A_5^+}$ =0.737, $A_6^* = \frac{A_6^-}{A_6^- + A_6^+}$ =0.645
$A_7^* = \frac{A_7^-}{A_7^- + A_7^+}$ =0.353, $A_8^* = \frac{A_8^-}{A_8^- + A_8^+}$ =0, $A_9^* = \frac{A_9^-}{A_9^- + A_9^+}$ =0.548

Table 19

For selection of problems find final scores and ranks.

Strategy	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
Final Scores	0.786	0.743	1	0.571	0.737	0.645	0.353	0	0.548
Ranks	2	3	1	6	4	5	8	9	7

$$A_3 > A_1 > A_2 > A_5 > A_6 > A_4 > A_9 > A_7 > A_8$$

So, A_3 is best.

Figure 1

Final scores and rankingof Fuzzy SAW methodforselection of winner of Twenty20 Cricket World Cup 2021

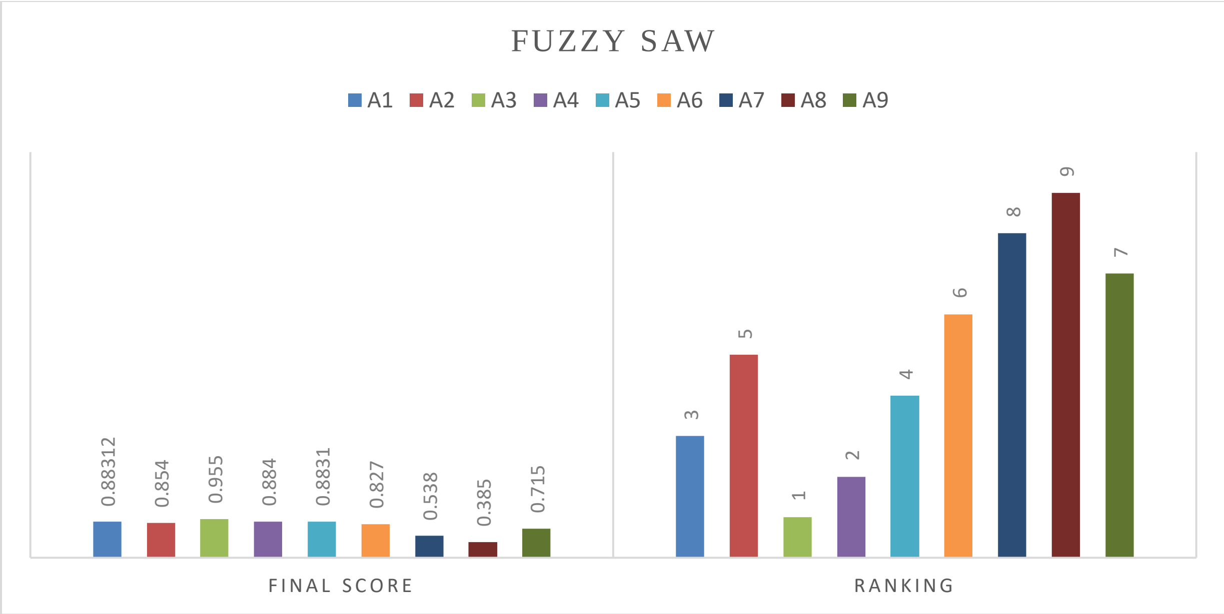


Figure 2

Final scores and ranking of modified Fuzzy SAW method for selection of winner of Twenty20 Cricket World Cup 2021

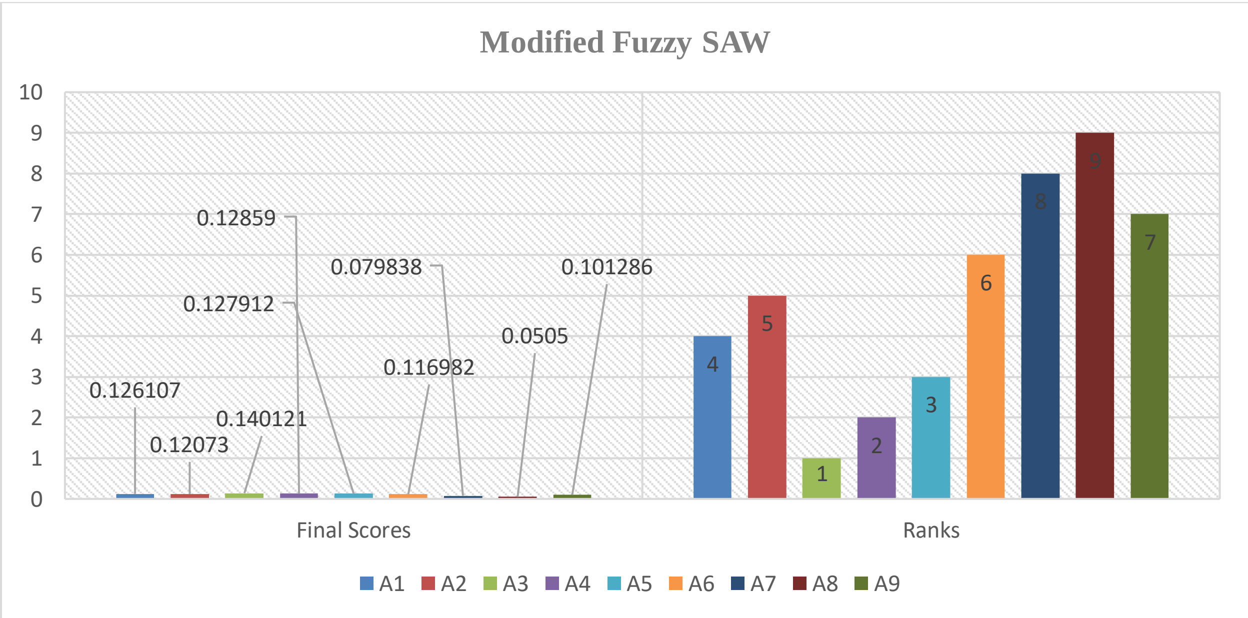


Figure 3

Final scores and ranking of Fuzzy TOPSIS method for selection of winner of Twenty20 Cricket World Cup 2021

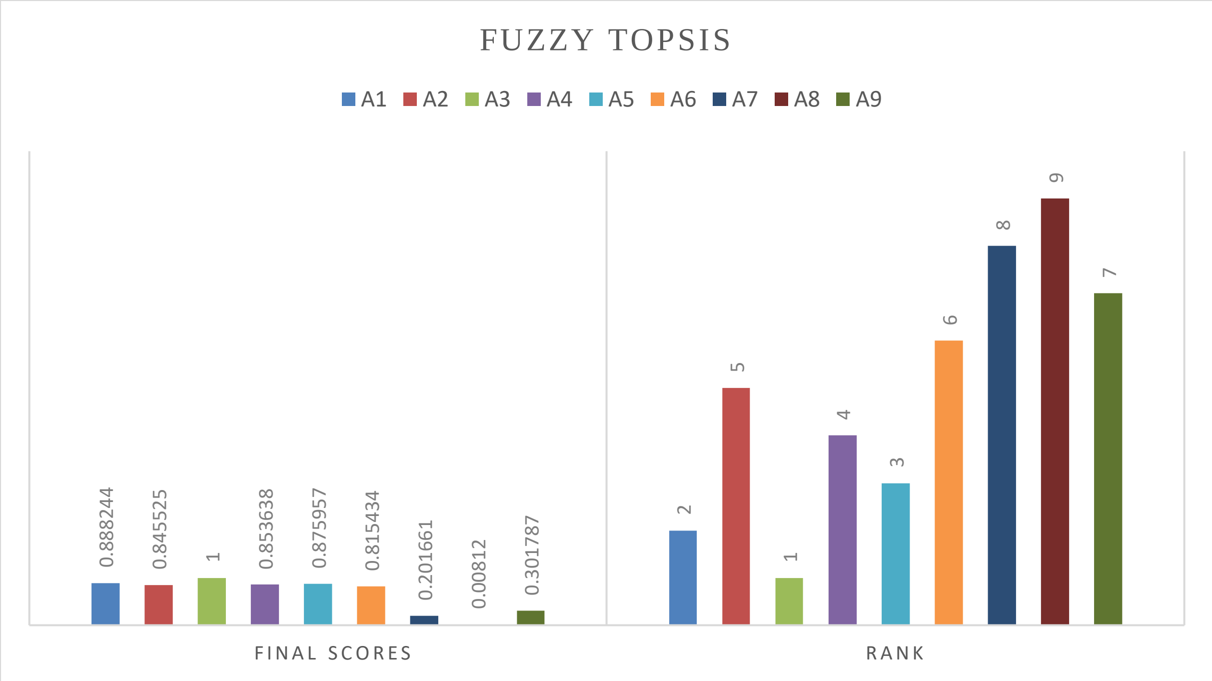
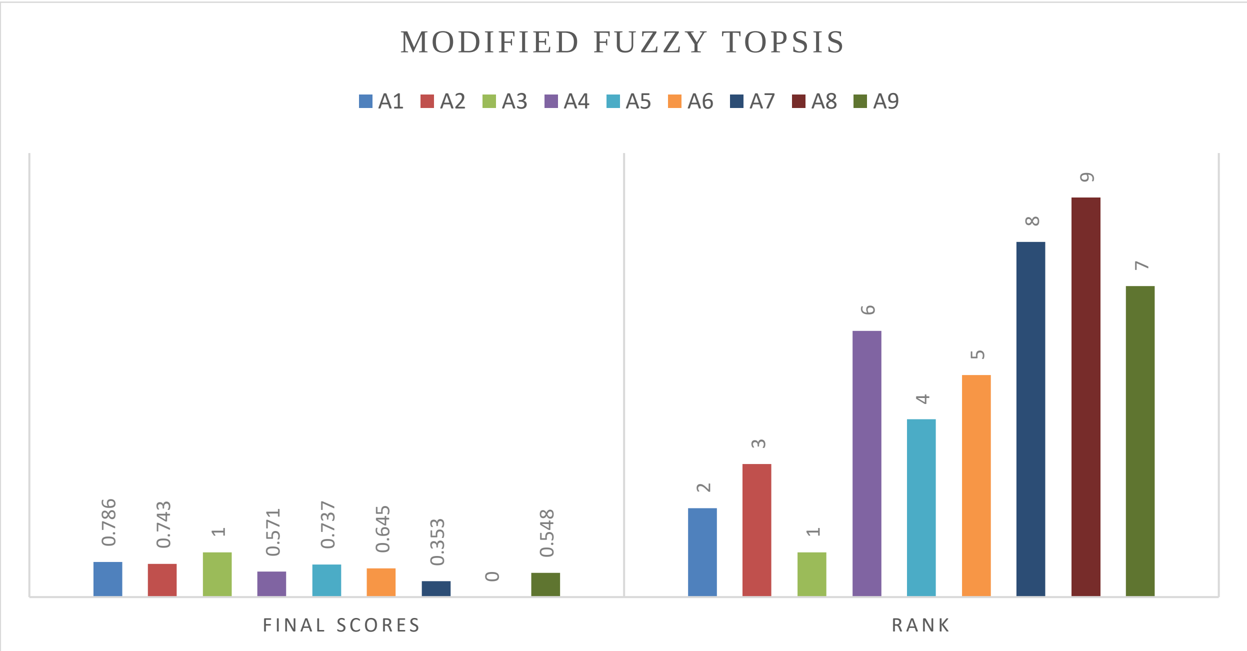


Figure 4

Final scores and ranking of modified Fuzzy TOPSIS method for selection of winner of Twenty20 Cricket World Cup 2021



Conclusion

The main goal of this paper was to predict the T20 CRICKET WORLD CUP 2021 based on current match statistics. Since the conditional result of a cricket match is related to many causes and unpredictable situation. Therefore, it is difficult responsibility to predict the exact and partial truth-based outcomes of cricket matches such research expects a multi criteria decision making approach, to solve this problem three existing methods are applied i.e. TOPSIS, Fuzzy TOPSIS, Fuzzy SAW method and the same problem is solved by Modified fuzzy TOPSIS, Modified Fuzzy SAW method. The result shows, India has the most chances of winning the T20 World-Cup 2021. The Team Bangladesh has the lowest chances of winning.

Reference

1. "How is T20 affecting cricket?".
2. ^ "Alex Tudor fears T20 is killing cricket's traditional skills".
3. ^ "T20 cricket has destroyed West Indies cricket: Sir Garfield Sobers - Firstpost". www.firstpost.com.
4. ^ "T20 has messed our cricket up - Lloyd".
5. ^ Gray, James (17 August 2017). "Why isn't Chris Gayle playing for West Indies against England? Test absence explained".
6. ^ "Gayle, Bravo, Pollard – Why Windies' Stars Will Skip India Series".
7. ^ White, Jim (1 June 2010). "Twenty20 will kill Test cricket within 20 years, says West Indian great Michael Holding" – via www.telegraph.co.uk.
8. ^ "I told Dravid not to retire - Ponting". ESPN Cricinfo. 25 August 2011. Retrieved 30 December 2018.
9. ^ "Benn stars in thrilling tie". cricinfo.com cricinfo.com. 26 December 2008. Retrieved 26 December 2008.
10. ^ "Vettori opposes Super Over". cricinfo.com cricinfo.com. 26 December 2008. Retrieved 5 February 2009.
11. ^ The Explainer (13 January 2009). "One1". cricinfo.com cricinfo.com. Retrieved 5 February 2009.
12. "Are T20 leagues making money?".
13. ^ "IPL television and broadcast rights sold for massive £1.97bn to Star India". The Guardian. 4 September 2017. Retrieved 18 February 2018.
14. ^ Weaver, Paul (25 May 2009). "Usman Afzaal gives Surrey winning start but absent fans fuel concerns". The Guardian. Retrieved 17 May 2012.
15. ^ "Sellout at WACA for Twenty20 match". ESPNcricinfo. 12 January 2005. Retrieved 17 May 2012.
16. "IPL 2017 Valuation". TOI. Retrieved 23 August 2017.
17. ^ "IPL world's 6th most attended league, Big Bash 9th: Report".
18. ^ "The lowdown on the major T20 leagues".
19. ^ World, Republic. "PSL 2020: League format, points, rules and all you need to know". Republic World. Retrieved 2 September 2020.
20. The solution of maintenance strategy selection problem by using modified fuzzy topsis for of material handling equipment : Fuzzy topsis for maintenance strategy selection problem, Sukkur IBA journal of computing and mathematical sciences, 3(2), 46-54, 2019