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AN INTERESTING MODEL-INDEPENDENT LOWER BOUND FOR THE PRICE OF SWISS RE MORTALITY BOND 2003

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ABSTRACT. In this paper, we introduce an interesting model-independent lower bound for the price of the Swiss Re Mortality Bond 2003 which was the first catastrophic mortality bond to be launched in the insurance market. This bond captures the behaviour of a well-defined mortality index to generate pay-offs for bondholders. Pricing of catastrophic mortality bonds is an interesting problem and no closed form solution can be found in the existing literature. In our approach, we express the pay-off of such a bond in terms of the pay-off of an Asian put option in a manner similar to [2] and present an efficient model-independent lower bound by making some very general assumptions. We carry out Monte Carlo simulations to estimate the bond price and illustrate the quality of the bound for a variety of models.

1. Introduction

Events that have the ability to bring the world to a standstill are referred to as catastrophes. Catastrophes inflate the claims of insurers and re-insurers to an unforeseen magnitude, thereby testing their capacity to settle the dues and at the same time eroding their reserves, even leading to instances of bankruptcy. Mankind has recently been a witness to the devastating COVID-19 pandemic. This catastrophic pandemic has highlighted the fact that even with all modern medical and technical advancements, human health and life is at stake as a small corona virus has wiped out almost 7 million human lives to date.

Long back in the mid-1990's post events such as Hurricane Andrew and North ridge Earthquake, 'Insurance Linked Securities' (ILS) called 'Catastrophic (Cat) Bonds' were floated in the market. This market has been growing ever since so much so that the year 2021 witnessed a record issuance of USD 12.8 billion in cat bonds to the world market (c.f. [68]). In fact in the year 2003, Swiss Re designed the first catastrophic mortality (CATM) bond also abbreviated as CMB. Since then as many as 16 CATM bonds have been issued. Most of these bonds are based on the value of a mortality index which if lies within a particular range, the principal and the coupons of the bond are paid in full; otherwise depending on its stretch beyond a stipulated range, erosion of the initial capital is triggered which is determined using a well-laid out formulation. Most of these bonds have been successful and have ended up being very lucrative investments for the investors

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since they are relatively free from impending market risks. We throw more light on the key features of these bonds in the next section.

Valuation of these bonds has been an intriguing puzzle for researchers and academic literature contains a flurry of research articles on this topic. Notable among the recent publications are the ones by [2] who utilize the theory of comonotonicity to construct model-independent bounds, [16] who introduce a mortality model that depicts the relevant pandemic effects on pricing mortality-linked securities (MLSs), using a threshold jump approach and [50] who consider correlation between mortality and interest rate to price excess mortality risk in a post pandemic era. We take a deeper dig at valuation of the CATM bonds in Section 3. Section 4 then presents the design of the Swiss Re Bond. Further the Section 5 unfolds the pay-off of the Swiss Re mortality bond in terms of the pay-off of an Asian put option. Section 6 showcases the put-call parity for this bond. In section 7, we propose a model independent lower bound. Section 8 presents numerical findings. The final section concludes the paper.

2. Key Features of CATM Bonds

The basic transaction structure of catastrophic mortality bonds has remained reasonably generic over most of the sixteen public transactions that have occurred to the end of 2023. The following figure shows the detailed structure.

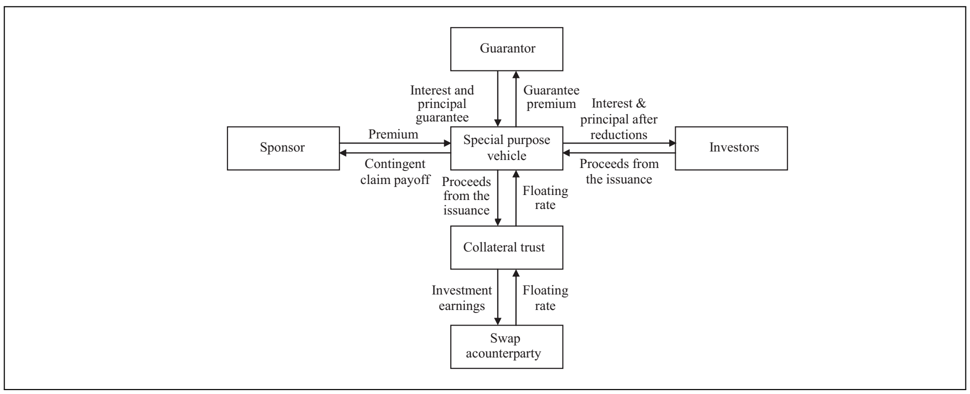

FIGURE 1. Basic Catastrophic Mortality Bond Structure (Source: [54])

The transaction involves three parties

- *The Ceding company or Sponsor*
- *Special Purpose Vehicle (SPV) or issuer*
- *Investors (generally large institutional buyers*[1]*)*

The transaction begins with the formation of a SPV which issues bonds to investors and invests the received capital in high quality securities such as government bonds

[1]Commonly pension funds with a view to hedge their position in terms of longevity and mortality

or AAA corporate bonds which are held in a trust account. The coupon paid to the buyer comprises of investment returns from this account plus the risk premium paid by the ceding company.

Embedded in these bonds is a call option that is triggered by a defined catastrophic event. Like the transaction structure, the contingent claim pay-off mechanism has remained more or less the same for all transactions. The key components of the contingent claim payoff mechanism are

- *principal amount*
- *coupon*
- *mortality index*
- *attachment/trigger point*
- *exhaustion point.*

The principal amount represents the maximum payoff that the sponsor can receive if the bond is triggered and this has typically ranged from U.S. $50 to U.S. $100 million per tranche. There are well defined attachment and exhaustion points and generally a very attractive periodic coupon to attract investors. The mortality index, attachment point, and exhaustion point determine whether the bond is triggered and if so, what percentage of the principal is paid. The mortality index is generally defined over a 2-calendar-year period in order to mitigate the chance that an influenza pandemic will be cut off by the end of the measurement period. This index is computed using general population mortality rates published by official public reporting sources weighted by age, gender and sometimes country ([61]). The weights are specified by the sponsor to broadly reflect their exposure (in terms of data availability and chance of a catastrophic occurrence) to an insured population and are fixed throughout the duration of the period over which the catastrophic bond provides coverage called the *risk period.* ([67]). As an example, the figure 2 represents the weight distribution of the very first CMB 'Vita I' issued by the Swiss Re in 2003.

The attachment and exhaustion points are expressed as a percentage of the mortality index at issuance. To date, the lowest attachment point has been 105 percent while the highest exhaustion point has been 150 percent. The reduction mechanism for the principal amount is triggered if the mortality index value exceeds the attachment point otherwise the full principal amount is returned to the investor at maturity. Once the attachment point is exceeded, the reduction in the principal amount increases linearly between the attachment and exhaustion points until the index exceeds the exhaustion point and the full principal is lost by the investor ([14]). For the VITA I bond discussed above the attachment and exhaustion points were 130 percent and 150 percent and the capital repayment or erosion phenomenon is shown in figure 3. Clearly these bonds are principal-at-risk instruments. The higher the trigger point is, the lower the chance that the event would actually happen and this would mean the lower the returns for the investor and vice-a-versa based on the principle of 'high risk-high reward'.

The choice of an index-based pay-off trigger is driven by investors' appetite for transparent, easy-to-understand, and hard-to-manipulate triggers ([77]) such as

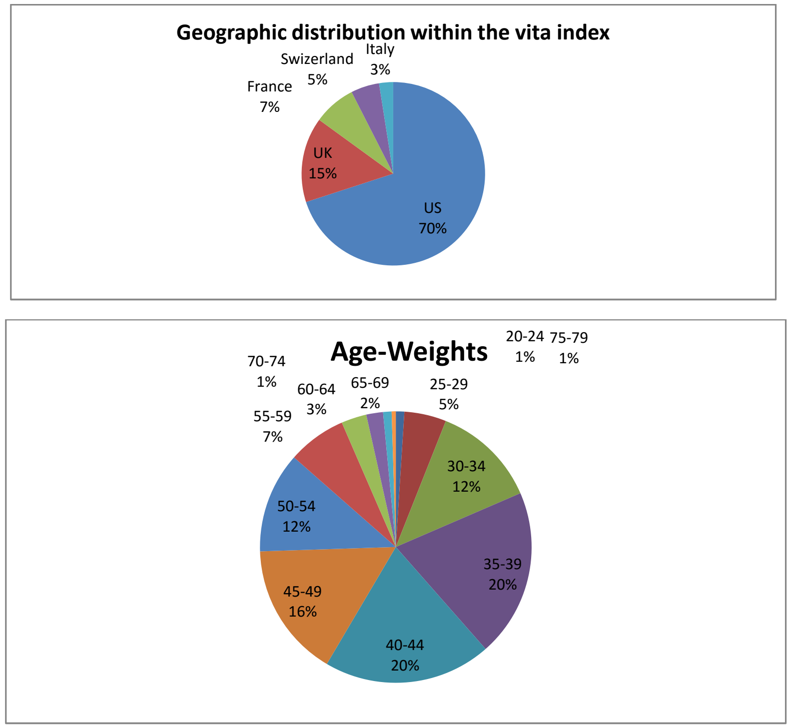

FIGURE 2. Weight Distribution of the VITA I Index

the ones based on indemnity[2]. Index-based pay-off triggers can be standardized more easily than their indemnity-based counterparts[3] and they reduce moral hazard because the sponsor still has an incentive to limit losses as the pay-offs are based on an independent metric rather than the sponsor's actual losses. Moreover, there is a reduction in adverse selection as pay-offs are based on publicly available data and there are few informational asymmetries to be exploited ([39];[13]). In the ensuing section we investigate the approaches that have been employed in academic literature in regards to valuation of CATM bonds.

3. Valuation

Given how mortality-linked bonds are structured, their pricing requires expertise in actuarial science (for assessing the impact of changes in mortality), econometrics (for modelLing the random evolution of mortality rates) and finance (for turning simulation results to prices). A lot of valuation approaches to price CMBs

[2]Indemnity: triggered by the issuer's actual losses, so the sponsor is indemnified, as if they had purchased traditional catastrophe reinsurance. If the layer specified in the cat bond is $100 million excess of $500 million, and the total claims add up to more than $500 million, then the bond is triggered [78].

[3]Queensgate and ALPS II are examples of life indemnity bonds issued by Swiss Re.

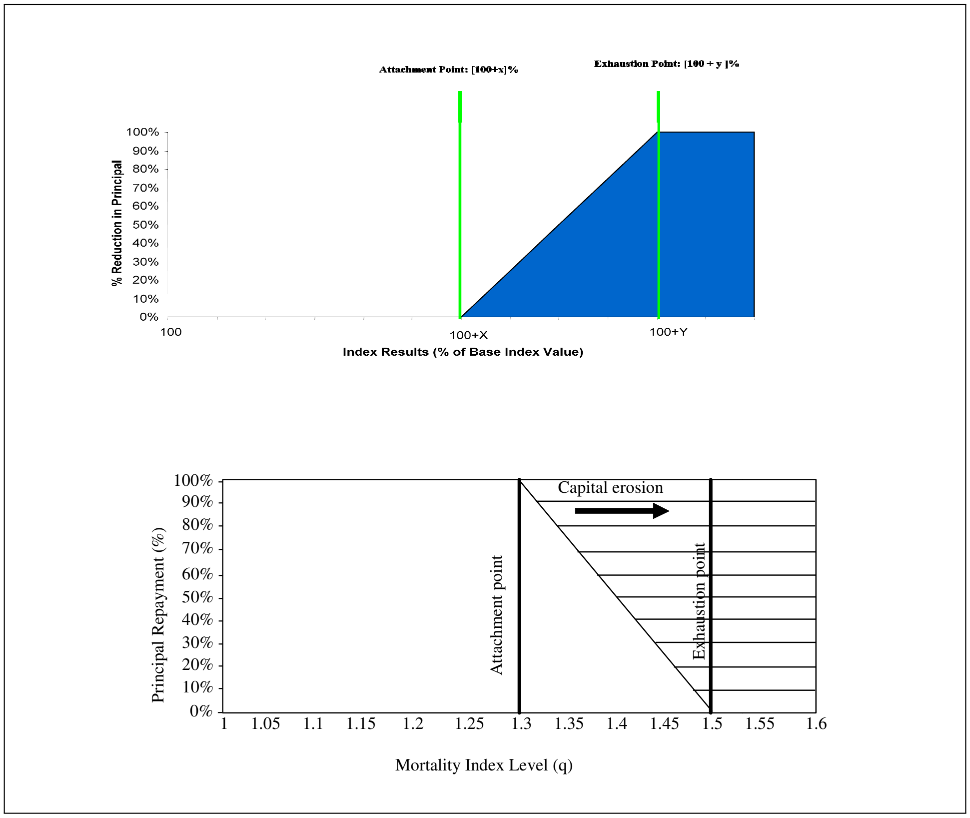

Figure 3. Capital Repayment/ Erosion for the Swiss Re 2003 CATM Bond (Source: [46] and [10])

have germinated and we discuss these in Chapter 3. To model extreme mortality risk and value CMBs, researchers have devised a number of stochastic models that incorporate jump effects. These models include some superb contributions by [4], [8], [17], [20], [18], [19, 22], [27, 26], [29], [38], [52], [53] and [81]. Various features of mortality jumps in terms of its occurrence and severity have been investigated in great depth. For example, [18] and [19] use independent Bernoulli distributions, [26] employed Poisson jump counts while [52] considered a discrete-time Markov chain.

Moreover, the first generation of econometric models for valuing catastrophic mortality bonds are univariate, modelling one population at a time. Models belonging to this category encompass those that were proposed by [18], [26], [29] and [55]. Although these models capture characteristics such as skewness and leptokurtosis, they are unable to accommodate the potential static- and cross-correlations among the mortality dynamics of different populations. Therefore, these models may not be efficient enough to price most of catastrophic mortality bonds, including Swiss Re's Vita I, which are linked to the mortality experiences of multiple populations. The second generation of models are multivariate, modelling all populations in question simultaneously. Models that fall into this category have been proposed by [81] who applied a combination of univariate time-series models with correlated innovations, [53] who employed a model with a common jump effect and correlated idiosyncratic risks, [21] who considered a combination of uni-variate GARCH models and a factor copula, [74] who linked uni-variate ARMA models with a dynamic copula and [75] who propose a DCC-GARCH model, in which the correlations are captured within the model structure (rather than externally through a copula) and are permitted to vary over time.

As the MLS market is incomplete, it is not possible to find a unique pricing measure. However the fact that the market is arbitrage-free, allows us to lay hands on at least one risk-neutral measure, whicht can then be utilized to obtain fair prices of mortality contingent securities such that no matter what the choice of such a measure is, the pricing is done in a model independent way. We exploit this fact and work in a model-independent atmosphere: that is, we refrain from assuming that the mortality evolution process behaves according to a given model, but aim to draw conclusions that hold under any model. This is in contrast to the standard approach to pricing mortality contingent products which is to postulate a model and to determine the price of the underlying as the suitably discounted risk neutral expectation of the pay-off under that model. A major drawback of this approach is that no model has the ability to capture the real world behaviour of MLSs fully, thus exposing the entire procedure to model risk.

4. Design of the Swiss Re Bond

As hinted in the introduction, the financial capacity of the life insurance industry to pay catastrophic death losses from natural or man-made disasters is limited. To expand its capacity to pay catastrophic mortality losses, Swiss Re obtained about 400 million in coverage from institutional investors in lieu of its first pure mortality security. The reinsurance giant issued a three year bond in December 2003 with maturity on January 1, 2007. To carry out the transaction, Swiss Re set up a special purpose vehicle (SPV) called Vita Capital Ltd. This enabled the corresponding cash flows to be kept off Swiss Re's balance sheet. The principal is subject to mortality risk which is defined in terms of an index q_{t_i} in year t_i. This mortality index was constructed as a weighted average of mortality rates (deaths per 100,000) over age, sex (male 65% and female 35%) and nationality (US 70%, UK 15%, France 7.5%, Italy 5% and Switzerland 2.5%) and is given below.

$$q_{t_i} = \sum_j C_j \sum_k A_k \left(G^m q^m_{k,j,t_i} + G^f q^f_{k,j,t_i} \right) \tag{4.1}$$

where q^m_{k,j,t_i} and q^f_{k,j,t_i} are the respective mortality rates (deaths per 100,000) for males and females in the age group k for country j, C_j is the weight attached to country j, A_k is the weight attributed to age group k (same for males and females) and G^m and G^f are the gender weights applied to males and females respectively.

The Swiss Re bond was a principal-at-risk bond. If the index q_{t_i} ($t_i = 2004$, 2005 or 2006 for $i = 1, 2, 3$ respectively) exceeds K_1 of the actual 2002 level, q_0, then the investors will have a reduced principal payment. The following equation describes the principal loss percentage, in year t_i:

$$L_i = \begin{cases} 0 & \text{if } q_{t_i} \leq K_1 q_0 \\ \frac{(q_{t_i} - K_1 q_0)}{(K_2 - K_1) q_0} & \text{if } K_1 q_0 < q_{t_i} \leq K_2 q_0 \\ 1 & \text{if } q_{t_i} > K_2 q_0 \end{cases} \tag{4.2}$$

In particular, for the case of Swiss Re Bond, $K_1 = 1.3$ and $K_2 = 1.5$. In lieu of having their principal at risk, investors received quarterly coupons equal to the three-month U.S. LIBOR plus 135 basis points. There were 12 coupons in all with a coupon value of

$$CO_j = \begin{cases} \left(\frac{SP + LI_j}{4}\right) .C & \text{if } j = \frac{1}{4}, \frac{2}{4}, ..., \frac{11}{4}, \\ \left(\frac{SP + LI_j}{4} .C + X_T\right) & \text{if } j = 3, \end{cases} \tag{4.3}$$

where SP is the spread value which is 1.35%, LI_j are the LIBOR rates, $C = \$400$ million, $T = t_3$ and X_T is a random variable representing the proportion of the principal returned to the bondholders on the maturity date such that

$$X_T = C \left(1 - \sum_{i=1}^{3} L_i\right)^+, \tag{4.4}$$

where $\sum_{i=1}^{3} L_i$ is the aggregate loss ratio at t_3. However, there was no catastrophe during the term of the bond. The discounted cash flow (DC) of payments is given by

$$DC(r) = \sum_{i=1}^{12} \frac{CO_{\frac{i}{4}}}{\left(1 + \frac{r}{4}\right)^i} \tag{4.5}$$

where r is the nominal annual interest rate.

Further define

$$Y_T = -\int_0^T \rho(t)\, dt$$

where $\rho(t)$ is the US LIBOR at time t. As a result, the risk-neutral value at time 0 of the random principal returned at the termination of the bond is

$$P = \mathrm{E}_Q\left[e^{-Y_T} X_T\right]$$

where Q is the risk-neutral measure. However, under the assumption of independence of Y_T and X_T, this reduces to

$$P = \mathrm{E}_Q\left[e^{-Y_T}\right] \mathrm{E}_Q[X_T]$$

The conditions under which it is possible (or not) to transfer the independence assumption from the physical world measure $\mathbb{P}$ to Q have been discussed extensively in [33]. Henceforth, in this incomplete market, we choose to price under a risk neutral measure that preserves independence between market and mortality risks. In order to proceed, we represent $\mathrm{E}_Q\left[e^{-Y_T}\right]$ as e^{-rT}, which implies

$$P = e^{-rT} \mathrm{E}_Q[X_T] \tag{4.6}$$

where r is the risk-free rate of interest. In subsequent writing, we drop Q from the above expression.

5. The Principal Payoff of Swiss Re Bond as that of an Asian-type Put Option

In the same spirit as [2], we can write X_T given in (4.4) in a more compact form similar to the pay-off of the Asian put option as shown below:

$$X_T = D\left(q_0 - \sum_{i=1}^{3} 5\left(q_{t_i} - 1.3q_0\right)^+\right)^+ \tag{5.1}$$

with

$$D = \frac{C}{q_0} \tag{5.2}$$

and the strike price equal to q_0. For the sake of simplicity, we use q_i in place of q_{t_i} and define

$$S_i = 5\left(q_i - 1.3q_0\right)^+ \tag{5.3}$$

and

$$S = \sum_{i=1}^{3} S_i \tag{5.4}$$

Using (5.3)-(5.4) in (5.1) and plugging the result into (4.6), we have:

$$P = De^{-rT}\mathrm{E}\Big[\left(q_0 - S\right)^+\Big] \tag{5.5}$$

It is naturally assumed that the inequalities $S \geq q_0$ almost surely (a.s.) and $S \leq q_0$ a.s. do not hold, otherwise the problem has a trivial solution. This means that $q_0 \in \left(F_S^{-1+}(0), F_S^{-1}(1)\right)$, where as in [31], F_X^{-1} is the generalized inverse of the cumulative distribution function (cdf), i.e.,

$$F_X^{-1}(p) = \inf\{x \in \mathbb{R} | F_X(x) \geq p\}, \quad p \in [0,1] \tag{5.6}$$

and F_X^{-1+} is a more sophisticated inverse defined as

$$F_X^{-1+}(p) = \sup\{x \in \mathbb{R} | F_X(x) \leq p\}, \quad p \in [0,1]\,. \tag{5.7}$$

Our interest lies in the calculation of reasonable bounds for P. In order to obtain a lower bound for P, we consider the call counterpart of the pay-off of Swiss Re Bond rather than (5.5). We nomenclate this pay-off as P_1, i.e., we have

$$P_1 = De^{-rT}\mathrm{E}\Big[\left(S - q_0\right)^+\Big] \tag{5.8}$$

We then exploit the put-call parity for Asian options to achieve the bounds for the pay-off in question.

6. Put-Call Parity for the Swiss Re Bond

We now derive the put-call parity relationship for the Swiss Re Bond. For any real number a, we have:

$$(a)^{+} - (-a)^{+} = a \tag{6.1}$$

So we obtain

$$e^{-rT}\left(\sum_{i=1}^{3} S_i - q_0\right)^{+} - e^{-rT}\left(q_0 - \sum_{i=1}^{3} S_i\right)^{+} = e^{-rT}\left(\sum_{i=1}^{3} S_i - q_0\right).$$

On taking expectations on both sides, we obtain

$$e^{-rT}\mathrm{E}\left[\left(\sum_{i=1}^{3} S_i - q_0\right)^{+}\right] - e^{-rT}\mathrm{E}\left[\left(q_0 - \sum_{i=1}^{3} S_i\right)^{+}\right] = e^{-rT}\mathrm{E}\left[\sum_{i=1}^{3} S_i - q_0\right].$$

Finally, on multiplying by D and expanding the definition of S_i, we have

$$P_1 - P = De^{-rT}\mathrm{E}\left[\sum_{i=1}^{3} 5\left(q_i - 1.3q_0\right)^{+} - q_0\right]$$

$$\Rightarrow P_1 - P = De^{-rT}\left[5\sum_{i=1}^{3} e^{rt_i} C\left(1.3q_0, t_i\right) - q_0\right], \tag{6.2}$$

where $C(K, t_i)$ depicts the price of a European call on the mortality index with strike K, maturity t_i and current mortality value q_0. As in [2], we note that this option would be in-the-money if the mortality index is greater than $1.3q_0$ which is the trigger level of Swiss Re bond. Clearly, such instruments are not available for trading in the market at present. But a more comprehensive life market is developing and we feel such securities will soon be introduced (c.f. [2], [12] and [11]). The pay-off structures, i.e. the design of the issued securities and the mortality contingent payments should be developed to appear attractive to investors and the re-insurer. Although, the Swiss Re bond was fully subscribed and press reports show that investors were quite satisfied with it (e.g. *Euroweek*, 19 December 2003), the market for mortality linked securities still needs innovations such as vanilla options on mortality index to provide flexible hedging solutions. Investors of the Swiss Re bond included a large number of pension funds as they could view this bond as a powerful hedging instrument. The underlying mortality risk associated with the bond is correlated with the mortality risk of the active members of a pension plan. If a catastrophe occurs, the reduction in the principal would be offset by reduction in pension liability of these pension funds. Moreover, the bond offers a considerably higher return than similarly rated floating rate securities (c.f. [10]). In a manner similar to [3], we feel the success of the life market hinges upon flexibility. As a result, such option-type structures enable re-insurer to keep most of the capital while at the same time being hedged against catastrophic mortality situation. [27] present an interesting note on the trigger level of $1.3q_0$ in context of 2004 tsunami in Asia and Africa. A mortality option of the above type would become extremely useful in such a case. [70] and [23] decompose the terminal pay-off of the Swiss Re bond into two call options.

Equation (6.2) gives the required put-call parity relation between the Swiss Re mortality bound and its call counterpart. Define

$$G = De^{-rT}\left[5\sum_{i=1}^{3} e^{rt_i} C\left(1.3q_0, t_i\right) - q_0\right]. \tag{6.3}$$

Clearly, if we bound P_1 by bounds l_1 and u_1, then the corresponding bounds for the Swiss Re mortality bond are as follows

$$\left(l_1 - G\right)^+ \leq P \leq \left(u_1 - G\right)^+. \tag{6.4}$$

7. An Interesting Lower Bound SWLB$_t^{(1)}$

We now proceed to work out an interesting lower bound for the terminal value of the principal paid in the Swiss Re Bond. For this, we first calculate bounds for the following Asian-type call option

$$P_1 = De^{-rT}\mathrm{E}\left[\left(\sum_{i=1}^{n} S_i - q_0\right)^+\right] \tag{7.1}$$

with $T = t_n$ and $n = 3$. The interval $[0, T]$ consists of the monitoring times $t_1, t_2, ..., t_{n-1}$. The undercurrent of the theory presented in this section is the paper by [1]. The following inequality holds for every random variable Y and every constant c

$$\mathbf{E}\left[a^+\right] \geq \mathbf{E}\left[a\mathbb{1}_{\{Y \geq c\}}\right]. \tag{7.2}$$

Motivated by [1], we choose $a = \left(\sum_{i=1}^{n} S_i - q_0\right)$ and $Y = q_t$, where they make an appropriate choice for t later on. This leads to

$$P_1 \geq De^{-rT}\mathbf{E}\left[\left(\sum_{i=1}^{n} S_i - q_0\right)\mathbb{1}_{\{q_t \geq c\}}\right]. \tag{7.3}$$

This reduces to:

$$P_1 \geq De^{-rT}\mathbf{E}\left[\sum_{i=1}^{n} S_i \mathbb{1}_{\{q_t \geq c\}} - q_0 \mathbb{1}_{\{q_t \geq c\}}\right]. \tag{7.4}$$

Now, again utilizing (7.2) we choose: $a = 5\left(q_i - 1.3q_0\right)$ and $Y = q_t$ so that (7.4) along with the definition of S_i in (5.3) yields:

$$P_1 \geq De^{-rT}\mathbf{E}\left[\sum_{i=1}^{n} 5\left(q_i - 1.3q_0\right)\mathbb{1}_{\{q_t \geq c\}} - q_0 \mathbb{1}_{\{q_t \geq c\}}\right]. \tag{7.5}$$

We then split the first sum into two parts at $j = \min\{i : t_i \geq t\}$ and condition the second part on the information available up to time t depicted by $\mathcal{F}_t$ so as to yield

$$P_1 \geq De^{-rT}\left(5\sum_{i=1}^{j-1}\mathbf{E}\left[q_i \mathbb{1}_{\{q_t\geq c\}}\right] + 5\sum_{i=j}^{n}\mathbf{E}\left[\mathbb{1}_{\{q_t\geq c\}}e^{r(t_i-t)}q_t\right]\right.$$
$$\left. - 6.5q_0\sum_{i=1}^{n}\mathbf{P}\left[q_t\geq c\right] - q_0\mathbf{P}\left[q_t\geq c\right]\right), \qquad (7.6)$$

where in the last equation, in the second term, we utilize the fact that discounted asset prices are martingales. We further modify the second term as follows.

$$\begin{aligned}\mathbf{E}\left[\mathbb{1}_{\{q_t\geq c\}}e^{r(t_i-t)}q_t\right] &= \mathbf{E}\left[\mathbb{1}_{[q_t\geq c]}e^{rt_i}e^{-rt}\left(q_t-c+c\right)\right]\\ &= e^{rt_i}e^{-rt}\mathbf{E}\left[\mathbb{1}_{\{q_t\geq c\}}\left(q_t-c\right)\right] + ce^{r(t_i-t)}\mathbf{P}\left[q_t\geq c\right]\\ &= e^{rt_i}e^{-rt}\mathbf{E}\left[\left(q_t-c\right)^+\right] + ce^{r(t_i-t)}\mathbf{P}\left[q_t\geq c\right]\\ &= e^{rt_i}C\left(c,t\right) + ce^{r(t_i-t)}\mathbf{P}\left[q_t\geq c\right]. \qquad (7.7)\end{aligned}$$

where $C(c,t)$ denotes the price of a European call on the mortality index with strike c, maturity t and current mortality value q_0. Putting equation (7.7) in equation (7.6), we obtain

$$P_1 \geq De^{-rT}\left(5\sum_{i=1}^{j-1}\mathbf{E}\left[q_i\mathbb{1}_{\{q_t\geq c\}}\right] + 5\sum_{i=j}^{n}e^{rt_i}C\left(c,t\right)\right.$$
$$\left. -\mathbf{P}\left[q_t\geq c\right]\left(q_0\left(1+6.5n\right) - 5c\sum_{i=j}^{n}e^{r(t_i-t)}\right)\right). \qquad (7.8)$$

Further, we assume that q_i and $\mathbb{1}_{\{q_t\geq c\}}$ are non-negatively correlated for $t > t_i$

$$\Rightarrow \mathrm{Cov}\left(q_i,\ \mathbb{1}_{\{q_t\geq c\}}\right) \geq 0 \Rightarrow \mathbf{E}\left[q_i\mathbb{1}_{\{q_t\geq c\}}\right] \geq \mathbf{E}\left[q_i\right]\mathbf{E}\left[\mathbb{1}_{\{q_t\geq c\}}\right].$$

Under the assumption of the existence of an Equivalent Martingale Measure (EMM), Q, the discounted mortality process is a martingale, so that

$$\mathrm{E}\left[q_t\right] = q_0e^{rt}. \qquad (7.9)$$

Using equation (7.9) we can bound the first term in equation (7.8) from below as follows.

$$\mathbf{E}\left[q_i\mathbb{1}_{\{q_t\geq c\}}\right] \geq q_0e^{rt_i}\mathbf{P}\left[q_t\geq c\right]$$

and this finally yields:

$$P_1 \geq De^{-rT}\left(5\sum_{i=j}^{n}e^{rt_i}C\left(c,t\right) - \mathbf{P}\left[q_t\geq c\right]\left(q_0\left(1+6.5n\right) - 5q_0\sum_{i=1}^{j-1}e^{rt_i} - 5c\sum_{i=j}^{n}e^{r(t_i-t)}\right)\right). \qquad (7.10)$$

Clearly, we have

$$C\left(c,t\right) = e^{-rt}\mathbf{E}\left[\left(q_t-c\right)^+\right] = e^{-rt}\mathbf{E}\left[\mathbb{1}_{\{q_t\geq c\}}\left(q_t-c\right)\right],$$

which finally leads to:

$$C\left(c,t\right) = e^{-rt}\mathbf{E}\left[\mathbb{1}_{\{q_t\geq c\}}q_t\right] - ce^{-rt}\mathbf{P}\left[q_t\geq c\right].$$

Let $g(.)$ denote the probability density function (p.d.f.) of the mortality index q_t. Then, we can write the above equation as

$$C(c,t) = e^{-rt}\left[\int_c^\infty x\, g(x)\, dx - c\int_c^\infty g(x)\, dx\right]. \tag{7.11}$$

Differentiating $C(c,t)$ w.r.t c, using Leibnitz rule for differentiation under the integral sign, since the limit involves c, we obtain

$$\frac{\partial}{\partial c}C(c,t) = e^{-rt}\left[-c\, g(c) - \mathbf{P}\left[q_t \geq c\right] + c\, g(c)\right]$$

$$\Rightarrow \mathbf{P}\left[q_t \geq c\right] = -e^{rt}\frac{\partial}{\partial K}C(K,t)\Big|_{K=c} =: -e^{rt}C_K(c,t)$$

Substituting $\mathbf{P}[q_t \geq c]$ in equation (7.10) and rearranging the terms, we achieve

$$P_1 \geq 5De^{-rT}\sum_{i=j}^{n} e^{rt_i}\left(C(c,t) + C_K(c,t)\left(\frac{(0.2+1.3n)\, q_0 - \sum_{i=1}^{j-1} e^{rt_i} q_0}{\sum_{i=j}^{n} e^{r(t_i - t)}} - c\right)\right). \tag{7.12}$$

Now define:

$$\tilde{c}_t = \frac{(0.2+1.3n)\, q_0 - \sum_{i=1}^{j-1} e^{rt_i} q_0}{\sum_{i=j}^{n} e^{r(t_i - t)}}. \tag{7.13}$$

Clearly, the right-hand side would be maximal if $c = \tilde{c}_t$ is given by (7.13). Hence, the optimal lower bound for the Asian-type call option is given by:

$$P_1 \geq 5De^{-rT}\max_{0\leq t\leq T} C\left(\tilde{c}_t, t\right)\sum_{i=j}^{n} e^{rt_i} =: \mathrm{lb}_t^{(1)} \tag{7.14}$$

where $c = \tilde{c}_t$ is given by (7.13) and $j = \min\{i : t_i \geq t\}$.

The existence of $\mathrm{lb}_t^{(1)}$ hinges upon the assumption of non-negative correlation between q_{t_i} and $\mathbb{1}_{\{q_t \geq c\}}$ for $t > t_i$. Finally, in the light of put-call parity derived in section 6, the trivial lower bound for the Swiss Re mortality bond is given as

$$P \geq \left(\mathrm{lb}_t^{(1)} - G\right)^+ =: \mathrm{SWLB}_t^{(1)}. \tag{7.15}$$

where G is defined in (6.3).

8. Performance of $\mathbf{SWLB}_t^{(1)}$

We present below in tables that follow the values of the lower bound vis-a-vis the well-known Monte Carlo (MC) Estimates for the price of the Swiss Re bond for a variety of models.

In tables 1 and 2, we assume that the mortality evolution process $\{q_t\}_{t\geq 0}$ obeys the Black-Scholes model, specified by the following stochastic differential equation (SDE)

$$dq_t = rq_t dt + \sigma q_t dW_t.$$

In order to simulate a path, we will consider the value of the mortality index in the three years that form the term of the bond, i.e., $n = 3$. In fact we consider the time points as $t_1 = 1, ..., t_n = T = 3$. We invoke the following equation to generate the mortality evolution:

$$q_{t_j} = q_{t_{j-1}} \exp\left[\left(r - \frac{1}{2}\sigma^2\right)\delta t + \sigma\sqrt{\delta t} Z_j\right] \quad Z_j \sim N(0,1), \quad j = 1, 2, \ldots, n. \tag{8.1}$$

We highlight below the parameter choices in accordance with [52]. The value of the interest rate is varied in table 1 while table 2 experiments with the variation in the base value of the mortality index while assuming a zero interest rate. Parameter choices for tables 1 and 2 with t specified in terms of years are:

$$q_0 = 0.008453,\ T = 3,\ t_0 = 0,\ n = 3,\ \sigma = 0.0388.$$

Table 2 is followed by figures 1-3. While figures 1 and 2 depict comparisons between the bounds, figure 3 portrays the price bounds for the Swiss Re bond generated by the Black-Scholes model. We will let MC denote the Monte Carlo estimate for the Swiss Re bond.

Table 1 reflects that the relative difference ($= \frac{|bound - MC|}{MC}$) between the lower bound and the benchmark Monte Carlo estimate increases with an increase in the interest rate for a fixed value of the base mortality index q_0. This observation is echoed by figure 1. On the other hand, figure 2 depicts the difference between the Monte Carlo estimate of the Swiss Re bond and the derived bound. The absolute difference between the estimated price and the bounds increase as the value of the base mortality index is increased and then there is a switch and this gap begins to diminish. This observation is supported by the fact that an increase in the starting value of mortality increases the possibility of a catastrophe which leads to the washing out of the principal or in other words the option goes out of money.

In our next example, we assume that the mortality rate 'q' obeys the four-parameter transformed Normal (S_u) Distribution (for details see [43] and [44]) which is defined as follows

$$sinh^{-1}\left(\frac{q - \alpha}{\beta}\right) = x \sim N\left(\mu, \sigma^2\right), \tag{8.2}$$

where α, β, μ and σ are parameters ($\beta, \sigma > 0$) and $sinh^{-1}$ is the inverse hyperbolic sine function.

r	$\text{SWLB}_t^{(1)}$	MC	S.E. of M.C.
0.035	0.899130889163	0.899131338643	0.000007814868
0.030	0.913324024548	0.913324365180	0.000005483857
0.025	0.927447505803	0.927447582074	0.000003766095
0.020	0.941626342687	0.941626356704	0.000002549695
0.015	0.955935721003	0.955935715489	0.000001673442
0.010	0.970419124546	0.970419112046	0.000001032941
0.005	0.985101139986	0.985101142704	0.000000646744
0.000	0.999995778016	0.999995770298	0.000000405336

Table 1. $\text{SWLB}_t^{(1)}$ for the Swiss Re Mortality Bond under the Black-Scholes Model with $q_0 = 0.008453$ and $\sigma = 0.0388$ in accordance with [52].

q_0	$\text{SWLB}_t^{(1)}$	MC	S.E. of MC
0.008	0.999999915252	0.999999915033	0.000000052478
0.009	0.999821987950	0.999822630214	0.000003051524
0.010	0.978310383929	0.978782997810	0.000042738093
0.011	0.610962123857	0.652245039892	0.000090193709
0.012	0.040209770810	0.094677358603	0.000089559585
0.013	0.000000000000	0.001665407936	0.000011391823
0.014	0.000000000000	0.000002890238	0.000000379522

TABLE 2. $\text{SWLB}_t^{(1)}$ for the Swiss Re Mortality Bond under the Black-Scholes Model with $r = 0.0$ and $\sigma = 0.0388$ in accordance with [52].

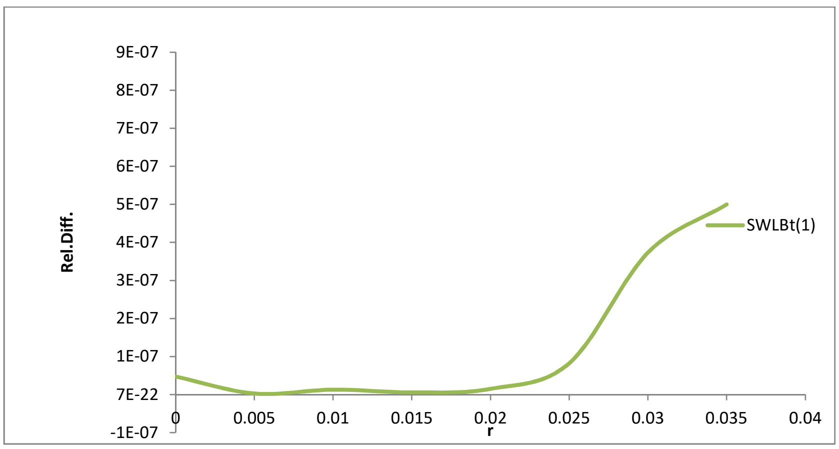

FIGURE 4. Relative Difference of $\text{SWLB}_t^{(BS)}$, $\text{SWUB}_t^{(BS)}$ and SWUB_1 w.r.t. MC estimate under Black-Scholes model

For table 3, we vary the interest rate as in table 1 and use the parameter set employed by [70]. The aforesaid authors use the mortality catastrophe model of [52] to generate the data and then utilize the quantile-based estimation of [66] to estimate the parameters of the S_u-fit. The initial mortality rate and time points are same as for tables 1 and 2. The following arrays present the values of the parameters for the three years 2004, 2005 and 2006 that were covered by the Swiss Re bond.

$$\alpha = [0.008399, 0.008169, 0.007905],\ \beta = [0.000298, 0.000613, 0.000904],$$

$$\mu = [0.70780, 0.58728, 0.58743] \text{ and } \sigma = [0.67281, 0.50654, 0.42218].$$

Finally in tables 4 and 5, we experiment with log gamma distribution by varying the interest rate in table 4 and the base mortality rate in the the latter. The parameters are chosen as in [23] who employ an approach similar to [70] outlined above with $q_0 = .0088$ but use maximum likelihood estimation to obtain the parameters of the fitted log gamma distribution. As before, the following arrays

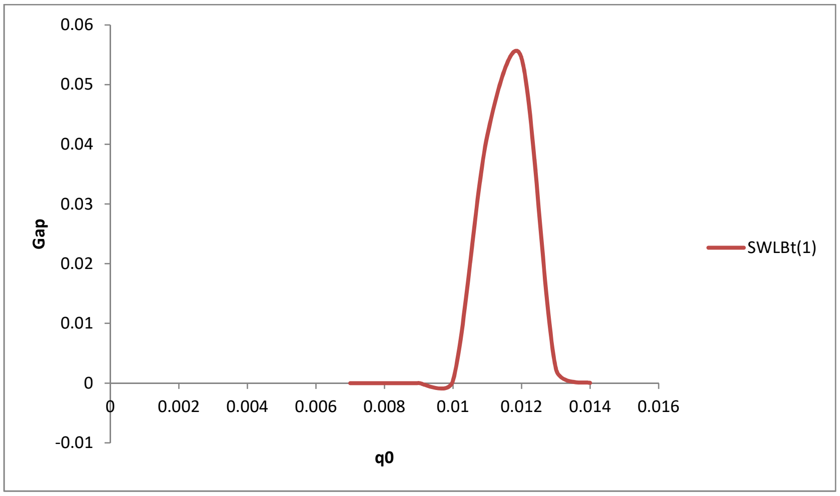

FIGURE 5. Comparison of different bounds under B-S model in terms of difference from MC estimate for r=0

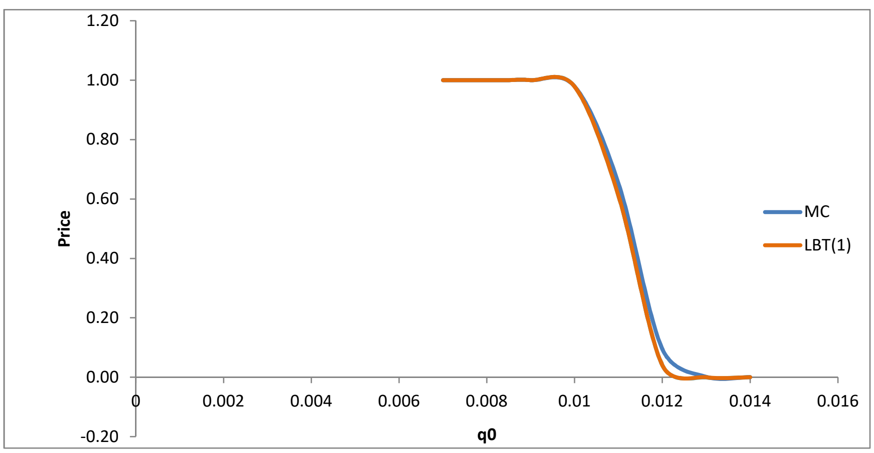

FIGURE 6. Price Bounds under Black-Scholes model for the parameter choice of Lin and Cox(2008) Model

present the year wise parameters

$$p = [61.6326, 64.2902, 71.8574], \ a = [0.0103, 0.0098, 0.0080],$$

$$\mu = [-5.2452, -5.4600, -5.7238] \text{ and } \sigma = [7.4 \times 10^{-5}, 9.5 \times 10^{-5}, 9.4 \times 10^{-5}].$$

Tables 4 and 5 clearly shows that even for non-normal universe, the bounds are extremely precise.

r	$SWLB_t^{(1)}$	MC	S.E.(MC)
0.035	0.88432143	0.88468962	0.00006349
0.030	0.90401002	0.90422765	0.00004987
0.025	0.92193552	0.92201394	0.00003804
0.020	0.93857698	0.93863396	0.00002794
0.015	0.95436972	0.95441569	0.00001956
0.010	0.96967776	0.96968765	0.00001352
0.005	0.98477952	0.98478917	0.00000859
0.000	0.99986838	0.99987622	0.00000513

TABLE 3. Lower Bounds and Upper Bound $SWUB_1$ for the Swiss Re Mortality Bond under the S_u distribution: $q_0 = 0.008453$ and parameter choice in accordance with [70].

r	$SWLB_t^{(1)}$	MC	S.E.(MC)
0.035	0.84849072	0.85408651	0.00049859
0.030	0.87384530	0.87815608	0.00044050
0.025	0.89725569	0.90050920	0.00038741
0.020	0.91898160	0.92103020	0.00034012
0.015	0.93928679	0.94092949	0.00028650
0.010	0.95842907	0.95947457	0.00024259
0.005	0.97664912	0.97748291	0.00020357
0.000	0.99417007	0.99466024	0.00016677

TABLE 4. Lower Bounds and Upper Bound $SWUB_1$ for the Swiss Re Mortality Bond under the transformed gamma distribution with $q_0 = 0.0088$ and parameter choice in accordance with [23].

q_0	$SWLB_t^{(1)}$	MC	S.E.(MC)
0.008	0.99976607	0.99978465	0.00003227
0.009	0.98914615	0.99003596	0.00023335
0.010	0.88804918	0.89137680	0.00077924
0.011	0.59608967	0.56844674	0.00128761
0.012	0.27104597	0.20822580	0.00105003
0.013	0.08274071	0.04612178	0.00052388
0.014	0.01270202	0.00673234	0.00019165

TABLE 5. Lower Bounds and Upper Bound $SWUB_1$ for the Swiss Re Mortality Bond under the transformed gamma distribution: $r = 0.0$ and parameter choice in accordance with [23].

9. Conclusions

Mortality forecasts are extremely important in the management of life insurers and private pension plans. Securitization and construction of mortality bonds has become an important part of capital market solutions. In the era prior to the launch of the Swiss Re bond in 2003, life insurance securitization was not designed to handle mortality risk.

This article proposes a model independent lower price bound for the Swiss Re mortality bond 2003. As stated in [29], an incomplete mortality market that has no arbitrage opportunities guarantees the existence of at least one risk-neutral measure termed the equivalent martingale measure Q that can be used for calculating the fair prices of mortality securities. We rely on this fact and devise this bounds for the mortality security in question without assuming any particular model. Model-specific bounds can then be achieved by plugging in the requisite models into the general bounds. The bound is extremely tight around the Monte Carlo values as can be compared from the the respective tables for all three models.

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