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SOLVING FUZZZY CRITICAL PATH WITH OCTAGONAL INTUITIONISTIC FUZZY NUMBER USING DIJKSTRA'S ALGORITHM

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Abstract. Network analysis is a technique which deals with the various sequences of activities concerned about the project and the completion of the project. During implementation of the project one may encounter various delay or vagueness while execution of the project.Critical path of the network gives an idea of minimum time one may expect to complete the project. Hence the importance of the critical path play a major role in network analysis. In this paper, the concept of finding fuzzy critical path using Octagonal fuzzy number is introduced. New Algebraic arithmetic of Octagonal fuzzy numbers is also discussed. A new method for finding the critical path of the problem is introduced with the help of Dijkstra's Algorithm and Octagonal fuzzy numbers. A suitable numerical examples are given to demonstrate the above methods.

1. Introduction

The amount of time needed to finish the project is directly proportional to the information contained in the network diagram. Typically, a project will consist of a number of activities, and in order to begin working on some activities, it will first be necessary to complete others activities. There is a possibility that some activities can be done separately from other activities. Network analysis is a method that can be used to ascertain the numerous activity sequences pertaining to a project as well as the amount of time necessary to finish the project. It has been utilized effectively in a broad spectrum of key management issues for the purpose of appraising particular kinds of initiatives.The Critical Path Method (CPM) and Program Evaluation and Review Techniques are examples of well-known applications of this methodology, both of which see widespread application (PERT). The length of the longest path from the beginning of the project to its conclusion is the shortest amount of time required to conclude the project. This is due to the fact that the activities in the network can be carried out in simultaneously. The network's most important route is the one that takes up the most time. The identification of critical activities along the critical route is the primary objective of CPM, as a result. On the other hand, as a result of the fuzziness of the time parameters involved in the problem, fuzzy CPM was developed. With the help of this fuzzy CPM, the unanticipated problem that can arise in the real world can be very well managed. The main purpose of CPM is

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thus to identify the critical activities on the critical path. However, the vagueness of the time parameters in the problem has led to the development of the fuzzy CPM for the past one decade. The vagueness that occur in the network problem that can be easily overcome by Fuzzy CPM. In [1] authors introduced the concept of FPERT using fuzzy number to represent activity duration in project network. In [2] Mon et al. introduced the concept of α- cut of each fuzzy duration they explored a linear combination of the duration bounds to represents the operation time of each activity and to determine the critical activities and paths by using the traditional (crisp) PERT technique.

Based on the α values different critical activities and different paths are obtained. Later in [3]Liberatore et al. proposed a straight forword method for applying fuzzy logic to assess uncertainity in critical path analysis. In [4] Chanas and Zielinski assumes that the operation time of each activity are represented either by a crisp value, interval or a fuzzy number and discussed the complexity of criticality. Chen and Huang [5], proposed a new model that combines fuzzy set theory with the PERT techniques to determine the critical degrees of activities and paths, lastest and earliest starting time and floats. Eventually the combination of fuzzy with PERT techniques has been emerged with the combination of new algorithms for finding CPM attracted many researchers for last five years. In [6] Ghoseri and Moghadam developed an algorithm to specifiy the critical path by the use of fuzzy sets, PERT techniques and Bellmann algorithm to specify the critical path and fuzzy earliest and latet starting time and floats of activities in the continuous fuzzy network. Thus a lot of papers are published in Fuzzy critical activites using above mentioned methods [7, 8, 9, 10].

Above all method decision - making method is the one of the method to find the best alternative in a set of feasible a alternatives. In this , several criteria's are taken into account called as multi-criteria decision making (MCDM) problems. It refers the ordering of the set of alternatives under conflicting criteria which are in complex in nature. Hence decision making often happens in fuzzy environment since the data available is in vague manner. Hence,in this stage one of the best method MCDM to prioritizing the set of alternatives under usually inconsistent criteria. Thus one of the MCDM method named as TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is presented. Initially [11] Hwang and Yoon presented TOPSIS method which deals about the concept of ideal alternative, for the optimum level of all available attributes. Further they classified as positive and negative ideal solution based on the best and worst attribute. Chen [12] had further introduced the extended TOPSIS method in fuzzy environment using Vertex method to find the distance between two triangular fuzzy numbers.

Interval valued intuitionist hesitant fuzzy based on TOPSIS method for MCDM presented by Joshi, Deepa [13, 14, 15]. In [16] Partha sarathy presented TOPSIS method based on the connection number of set pair analysis under interval- valued intuitionistic fuzzy environment has proposed. Later a Dual Hesitant Fuzzy Set (DHFS) has been considered a alternative form of Hesistant Fuzzy Set (HFS) and disagree with the effects of uncertainty in the collected data. The one of the drawbacks of HFS and DHFS is crisp - numbers are lies between 0 and 1 which are primarily used to repreent the membership and non-membership degree of an element of a particular set.In [15], Kalkati, Pankaj et al presented " Interval neutrosophic hesitant fuzzy Einstein Choquet integral operator" for multi criteria decision making has been discussed. Recently, Partha saradhi et al, have introduced Hesitant Fuzzy number for finding critical path problem using TOPSIS approach. They used new distance measure for evaluating distance for Triangular Hesitant Fuzzy set. In all the above approaches in finding the CPM authors used several fuzzy set such as triangular, hexagonal, neutrosophic and apply MCDM approch using TOPSIS method to find the CPM.

This paper is organized as follows. In section 2, some basic concepts and definition are given for fuzzy sets . In section 3 Octagonal Intutionistic Fuzzy set has been introduced and the basic arithmetic operations like addition, subtraction, multiplication with examples are discussed in section 4. In section 5 a proposed method for finding a critical path has been discussed using Dijiskra Algorithm. In Section 6 two examples are demonstrated to validate the proposed method and conclusion are discussed in section 7.

2. Preliminaries

This section provides with basic definition of fuzzy sets and its related topics.

Definition 2.1. Let X be a set. A fuzzy set $\bar{A}$ on X is defined to be a function $\mu_A : X \to [0,1]$ is a mapping called the membership value of x ∈ X in a fuzzy set $\bar{A}$

Definition 2.2. The fuzzy number $\bar{A}$ is a fuzzy if membership function satisfies i) A fuzzy set of the universe of discourse ii) $\bar{A}$ is normal if $x_i \in X$, $\mu_A(x) = 1$, ii) $\mu_A(x)$ is piecewise continuous.

Definition 2.3. A fuzzy number $\bar{A}$ is a normal octagonal fuzzy number denoted be $(c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8)$ which real numbers with membership function as

$$\mu_A(x)=\begin{cases} 0 & x \le c_1 \\ 0.5 \quad \frac{x-c_1}{0.5-c_1} & c_1 \le x \le c_2 \\ 0.5 - 0.5\frac{x-c_2}{c_3-c_2} & , c_2 \le x \le c_3 \\ \frac{x-c_3}{c_4-c_3} & , c_3 \le x \le c_4 \\ 1 & c_4 \le x \le c_5 \\ \frac{c_6-x}{c_6-c_5} & c_5 \le x \le c_6 \\ 0.5\frac{x-c_6}{c_7-c_6} & c_6 \le x \le c_7 \\ 0.5 - 0.5\frac{c_8-x}{c_8-c_7} & , c_7 \le x \le c_8 \\ 0 & x \ge c_8 \end{cases}$$

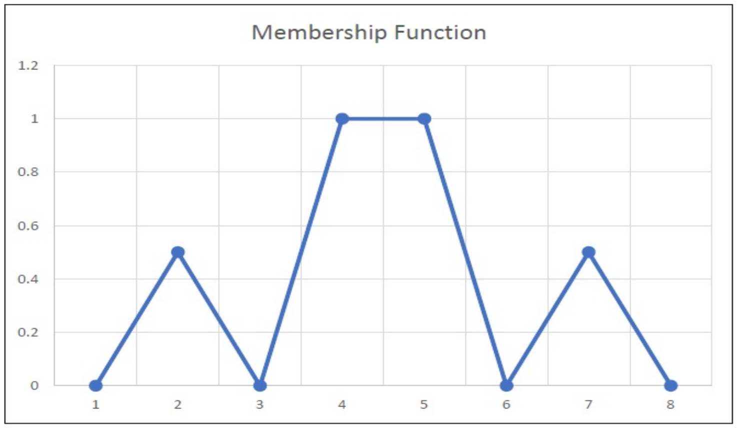

FIGURE 1. Octogonal Fuzzy Number

Definition 2.4. An Intuitionistic fuzzy set $\bar{I}$ of non-empty set X is defined as $\bar{I} = \{t, \mu_{\bar{I}}(t), \gamma_{\bar{I}}(t) : X \to [0,1]\}$, $\mu_{\bar{I}}(t) \to$ membership function, $\gamma_{\bar{I}}(t) \to$ a non-membership function such that $\mu(t), \gamma(t) : X \to [0,1]$ and $0 \leq \mu(t) + \gamma(t) \leq 1$ $\forall t \in X$

Definition 2.5. A subset B $= \{\langle t, \mu(t), \gamma(t)\rangle \, t \in X$ of the real line R is said to be Intuitionistic Fuzzy number, if i) $\exists x \in R, \ni \mu(t) = 1, \gamma(t) = 0$ ii) $\mu(t)$ is continuous from R $\to [0,1]$ iii) $0 \leq \mu(t) + \gamma(t) \leq 1 \; \forall t \in X$

3. OCTAGONAL INTUITIONISTIC FUZZY NUMBER

In this section, we discussed with notion of OCTAGONAL INTUTIONISTIC FUZZY NUMBER (OCINTFN) and their arithmetic operations.

Definition 3.1. An OCINTFN is $I_{OCT} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$, $\{a_1^1, a_2^2, a_3^3, a_4^4, a_5^5, a_6^6, a_7^7, a_8^8$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_1^1, a_2^2, a_3^3, a_4^4, a_5^5, a_6^6, a_7^7, a_8^8$ are real numbers

The membership function is $\mu_A(t)=$ $\begin{cases} 0 & x \leq a_1 \\ 0.5\frac{t-a_1}{0.5-a_1} & a_1 \leq t \leq a_2 \\ 0.5 - 0.5\frac{t-a_2}{a_3-a_2} & , a_2 \leq t \leq a_3 \\ \frac{t-a_3}{a_4-a_3} & , a_3 \leq t \leq a_4 \\ 1 & a_4 \leq t \leq a_5 \\ \frac{a_6-t}{a_6-a_5} & a_5 \leq t \leq a_6 \\ 0.5\frac{t-a_6}{a_7-a_6} & a_6 \leq t \leq a_7 \\ 0.5 - 0.5\frac{a_8-t}{a_8-a_7} & , a_7 \leq t \leq a_8 \\ 0 & t \geq a_8 \end{cases}$

The non membership function is $\gamma_A(t)=\begin{cases} 1 & x \leq a_1^1 \\ 0.5+0.5\frac{a_1^1-t}{0.5-a_1} & a_1^1 \leq t \leq a_2^2 \\ 0.5+0.5\frac{a_2^2-t}{a_2^3-a_3^3} & ,a_2^2 \leq t \leq a_3^3 \\ \frac{a_3^3-t}{a_3^3-a_4^4} & ,a_3^3 \leq t \leq a_4^4 \\ 0 & a_4^4 \leq t \leq a_5^5 \\ \frac{t-a_6^6}{a_5^5-a_6^6} & a_5^5 \leq t \leq a_6^6 \\ 0.5\frac{a_6^6-t}{a_6^6-a_7^7} & a_6^6 \leq t \leq a_7^7 \\ 0.5+0.5\frac{t-a_8^8}{a_7^7-a_8^8} & ,a_7^7 \leq t \leq a_8^8 \\ 1 & t \geq a_8^8 \end{cases}$

4. Basic Operations in OCINTFN

If $\bar{I}_{oct} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$, $(a_1^1, a_2^2, a_3^3, a_4^4, a_5^5, a_6^6, a_7^7, a_8^8)$, $\bar{J}_{oct} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$,$(b_1^1, b_2^2, b_3^3, b_4^4, b_5^5, b_6^6, b_7^7, b_8^8)$, be any two OCINTFN then

(i) Addition of OCINTFN

$\bar{I} + \bar{J} = (a_1 + b_2, a_2 + b_2, a_3 + b_3, a_4 + b_,4 a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8,)$ and $(a_1^1 + b_1^1, a_2^2 + b_2^2, a_3^3, b_3^3, a_4^4 + b_4^4, a_5^5 + b_5^5, a_6^6 + b_6^6, a_7^7 + b_7^7, a_8^8 + b_8^8)$

(ii) Subtraction of OCINTFN

$\bar{I} - \bar{j} = (a_1 - b_2, a_2 - b_2, a_3 - b_3, a_4 - b_,4 a_5 - b_5, a_6 - b_6, a_7 - b_7, a_8 - b_8,)$ and $(a_1^1 - b_1^1, a_2^2 - b_2^2, a_3^3 - b_3^3, a_4^4 - b_4^4, a_5^5 - b_5^5, a_6^6 - b_6^6, a_7^7 - b_7^7, a_8^8 - b_8^8)$

(iii) Multiplication of OCINTFN

$\bar{I} * \bar{J} = (a_1 * b_2, a_2 * b_2, a_3 * b_3, a_4 * b_,4 a_5 * b_5, a_6 * b_6, a_7 * b_7, a_8 * b_8,)$ and $(a_1^1 * b_1^1, a_2^2 * b_2^2, a_3^3 * b_3^3, a_4^4 * b_4^4, a_5^5 * b_5^5, a_6^6 * b_6^6, a_7^7 * b_7^7, a_8^8 * b_8^8)$

(iv) Division of OCINTFN

$\bar{I}/\bar{J} = (a_1/b_1, a_2/b_2, a_3/b_3, a_4/b_,4, a_5/b_5, a_6/b_6, a_7/b_7, a_8/b_8,)$
and $(a_1^1/b_1^1, a_2^2/b_2^2, a_3^3/b_3^3, a_4^4/b_4^4, a_5^5/b_5^5, a_6^6/b_6^6, a_7^7/b_7^7, a_8^8/b_8^8)$

Example 1 Consider the set $A_{oct} = \{(3, 4, 5, 6, 1, 2, 6, 7), (2, 1, 2, 3, 5, 6, 7, 8)\}$, $B_{oct}= \{(2, 1, 3, 4, 0, 1, 5, 4), (1, 2, 3, 4, 5, 6, 7, 8)\}$ be any two OCINTFN then
$A_{oct} + B_{oct} = (5, 5, 8, 10, 1, 3, 11, 11), (3, 3, 5, 7, 10, 12, 14, 16)$ are the two new fuzzy OCINTFN.
$A_{oct} - B_{oct} = (1, 3, 2, 2, 1, 1, 1, 3), (1, -1, -1, 0, 0, 0, 0, 0)$ are the two new fuzzy OCINFN. similarly we can easily define the multiplication and division operations as defined above.

5. Symbolic Representation

Let $\bar{E}S_i$ and $\bar{L}S_i$ be the earliest fuzzy event time, and the latest fuzzy time for event i, respectively. Functions that define the earliest starting times, latest starting times and floats in terms of fuzzy activity duration are convex, normal whose membership function are piecewise continuous hence the quantities such as earliest fuzzy event time $\bar{E}S_i$, the latest fuzzy event time $\bar{L}S_i$ are OCINTFN fuzzy numbers for an event i respectively. For basic computations, let us use the following notations:

(1) $A_{OCINTFN}(ij)$ = Activity of OCINTFN between event i and event j
(2) $E_{OCINTFN}(i)$ = Earliest occurrence event time i of OCINTFN
(3) $L_{OCINTFN}(j)$ = Latest occurrence event time j of OCINTFN
(4) $ES_{OCINTFN}(ij)$ = Earliest starting time from activity i to j of OCINTFN
(5) $EF_{OCINTFN}(ij)$ = Earliest finishing time from activity i to j of OCINTFN
(6) $LS_{OCINTFN}(ij)$ = Latest Starting time from activity i to j of OCINTFN
(7) $LF_{OCINTFN}(ij)$ = Latest Finishing time from activity i to j of OCINTFN
(8) $TF_{OCINTFN}(ij)$ = Total Float time of $A_{OCINTFN}(ij)$
(9) $E_{OCINTFN}(ij)$ = Estimated completion time.

6. Procedure for Fuzzy critical path Algorithm

Step 1 In project network, identify the OCINTFN activities.
Step 2 Establish precedence relationship of all fuzzy activities in terms of OCINTFN numbers.
Step 3 Draw a project network diagram with OCINTFN as fuzzy activity times.
Step 4 Find the expected time for the activity $a_0 - a_1$ using the following rule. Round off the expected time to the nearest largest integer.
Step 4 a) Consider the fuzzy activity time $(3, 7, 11, 15, 19, 24, 3, 7)$ for the path 1-2.
Step 4 b) Now 3 is assigned as 0, in (figure 1), 7 is assigned as 0.5, for 11, 15, 19, 24 apply mod 8 we get 3, 7, 3, 1.
Step 4 c) Thus finally we get as 0, 0.5, 0, 0.5, 0, 0, 0, 0.5
Step 4 d) Add all the numbers 0 + 0.5 + 0 + 0.5 + 0 + 0 + 0 + 0.5 = 1.5 which is the expected time the path 1-2
Step 4 e) Apply the above four steps for all the path. Thus we got the expected time for each path which was shown in table.
Step 5 At this stage, our project network has an associated integer calculated in Step 3 for any two adjacent nodes.
Step 6 Consider S and D as the source node and destination nodes in the network. Apply Dijkstra's algorithm to find the shortest path between the source node and the destination node. The path identified by the Dijkstra's algorithm is identified as the fuzzy intuitionistic critical path.

Example-1
consider the following network: The following table represents the calculation of Expected time in each activity of fuzzy project network in the numerical example and identify the critical path.

In this example 1, path $P2 : 1 - 2 - 4 - 7$ is identified as fuzzy critical path.
Example 2 The following table represents the calculation of Expected time in each activity of fuzzy project network in the numerical example and identify the critical path.

In this example 2, path $P4 : 1 - 3 - 6 - 7$ is identified as fuzzy critical path.

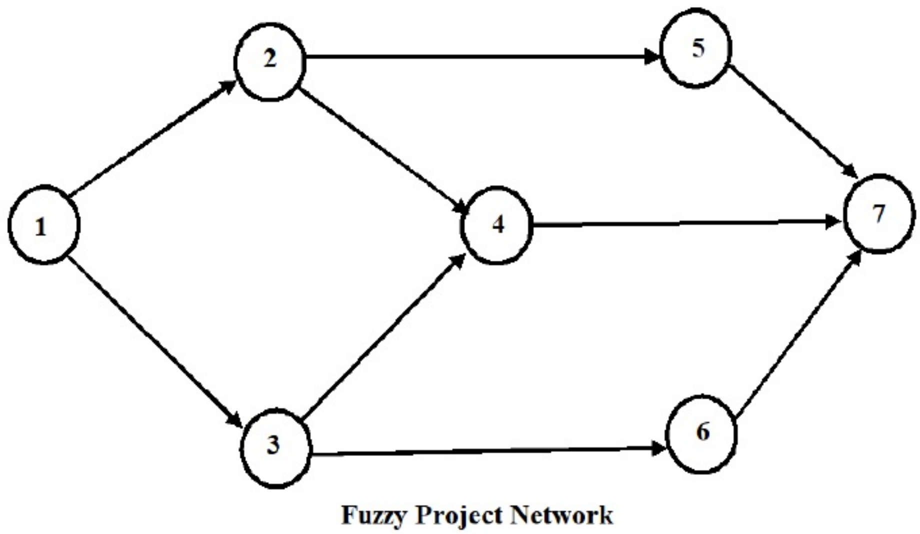

FIGURE 2. Network Diagram

Activity	Fuzzy Activity Time	Fuzzy Activity time converted into expected time
1-2	(3,7,11,15,19,24,3,7)	1.5
1-3	(3,5,7,9,10,12,3,5)	4
2-4	(11,14,17,21,25,30,11,14)	1.5
3-4	(3,5,7,9,10,12,3,5)	4
2-5	(5, 7,10,13,12,21,5,7)	5.5
3-6	(7,9,11,14,18,22,7,9)	1
4-7	(7,9,11,14,18,22,9,7)	1
5-7	(2,3,4,6,7,9,2,3)	2.5
6-7	(5,7,8,11,14,17,5,7)	3

TABLE 1. Fuzzy Activity time converted into Expected time

Path	Project completion time
P1: 1-2-5-7	9.5
P2: 1-2-4-7	4
P3: 1-3-4-7	9
P4: 1-3-6-7	8

TABLE 2. Project completion time using Dijikstra's algorithm

7. Conclusion

In this paper Dijkstra's Algorithm has been applied to a fuzzy project network to determine the critical path using several criteria. Octagonal Fuzzy number

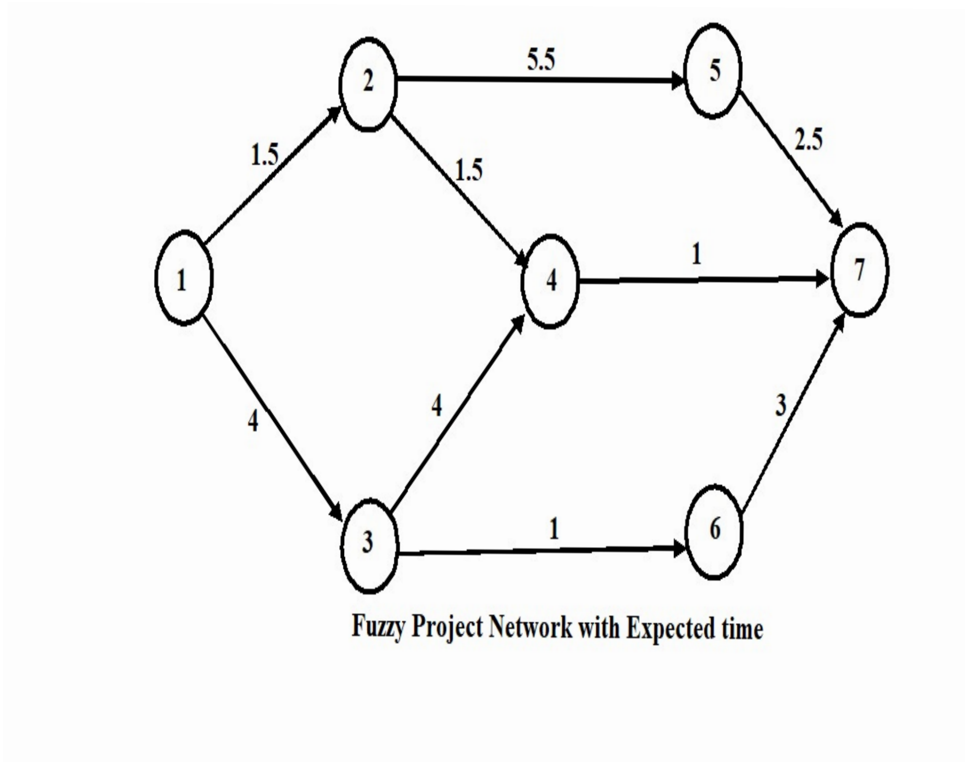

FIGURE 3. Expected time of a fuzzy network

Activity	Fuzzy Activity Time	Fuzzy Activity time converted into expected time
1-2	(13,27,14,17,21,27,13,17)	3
1-3	(23,15,27,49,30,32,23,52)	3
2-4	(11,24,37,24,27,32,17,19)	1
3-4	(13,15,27,39,44,22,23,30)	2
2-5	(15,27,20,33,27,31,25,35)	3
3-6	(17,19,11,24,19,26,17,19)	0.5
4-7	(17,29,13,19,28,27,19,7)	3.5
5-7	(12,23,14,26,17,19,12,13)	4
6-7	(15,27,18,21,24,27,18,17)	1.5

TABLE 3. Fuzzy Activity time converted into Expected time

have been used as fuzzy activity times, to find critical path of the project network. A new expected time has been proposed to select critical path using Dijkstra's Algorithm and octagonal fuzzy number as activity times. Two numerical examples related to this problem has provided to explain the procedure of the proposed method in determining critical path with different criteria.

Path	Project completion time
P1: 1-2-5-7	10
P2: 1-2-4-7	7.5
P3: 1-3-4-7	8.5
P4: 1-3-6-7	5

TABLE 4. Project completion time

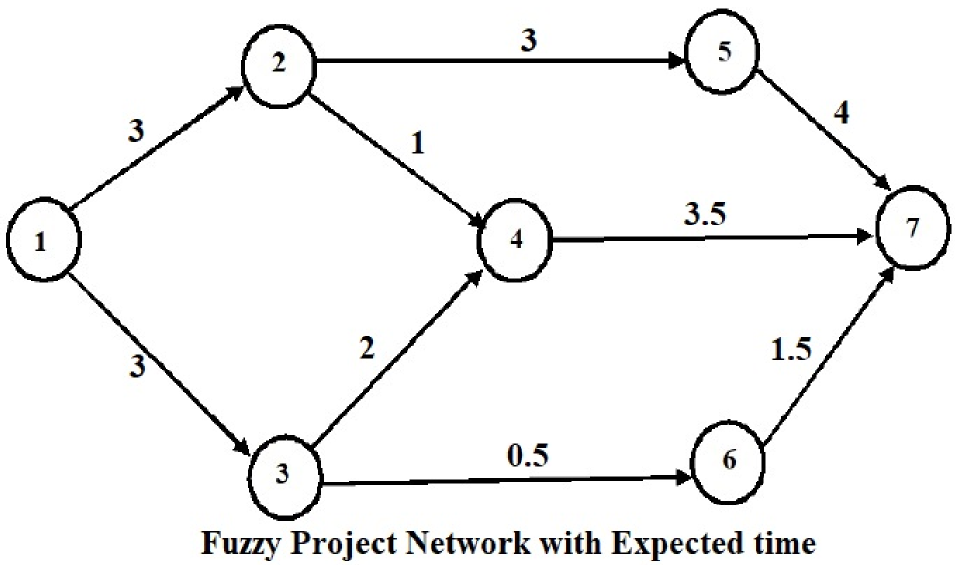

FIGURE 4. Expected time of a fuzzy network

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