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A SURVEY ON EXISTING CONDITIONS OF HAMILTONIAN GRAPHS

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Abstract. Most of the conditions for the existence of Hamiltonian graph trace their origin from the papers of Dirac (1952) and Ore (1960). In this paper, we have tried to collect conditions for the existence of Hamiltonian graphs.

1. Preamble

Let $G = (V, E)$ be a graph with minimum degree $\delta(G)$ and independence number $\alpha(G)$. For a vertex $v \in V(G)$, $d(v)$ and $N(v)$ denotes the degree of v and neighborhood of v in G, respectively. A number of sufficient conditions for a simple connected graph of finite order to be Hamiltonian have been proved. The two well known conditions for a graph G to be Hamiltonian are

- Dirac [19] Condition ($\delta(G) \geq \frac{n}{2}$).
- Ore's [47] condition (for any two nonadjacent vertices u and v, $d(u) + d(v) \geq n$).

In 1978, Bondy [9] proved that any simple graph G which satisfy Ore's condition also satisfy $Chvátal - Erdös$ condition. $Chvátal - Erdös$ theorem is the generalisation of Ore's theorem. Bondy [9] proved that every m-regular simple graph on $2m + 1$ vertices is Hamiltonian for $m > 0$. Ainouche and Christofides [3] derived sufficient conditions for a graph to be Hamiltonian. These conditions are stronger version of Bondy-Chvátal [10] conditions. Ainouche and Chrisofides [3] defined the following notations: Two $a - b$ paths in a graph G are compatible if the vertices belonging to both paths occur in the same order along the paths (when they are traversed from a to b). If μ is a given $a - b$ path in G, then $h^{\mu}_{ab}(G)$ be the maximum integer k such that G contains k internally vertex disjoint $a - b$ paths, each one compatible to μ. Let $\alpha_{ab}(G)$ be the maximum cardinality of an independent vertex set of G containing both a and b. If μ is a Hamiltonian $a - b$ path and $h^{\mu}_{ab} \geq \alpha_{ab}(G)$, then they proved that G has Hamiltonian circuit. Ainouche and Chrisofides [3] also defined $l_{ab}(G) = |\Gamma(a) \cap \Gamma(b)|$, where $\Gamma(i)$ is the set of neignbouring vertices of vertex i and if a and b are two non-adjacent vertices of a graph G such that $\alpha_{ab}(G) \leq l_{ab}(G)$ then G is Hamiltonian if and only if $G + ab$ is Hamiltonian. They have also proved the Bondy-Chvátal [10] conditions. Sir William Rowan Hamilton in 1859 suggested a class of a graph in which there exist

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a spanning cycle containing every vertex of dodecahedron. If a graph contains this spanning cycle, then the graph is said to be Hamiltonian graph and the cycle is called Hamiltonian cycle (circuit). After removing an edge from a Hamiltonian cycle, we are left with a path, known as Hamiltonian path. If exists a Hamiltonian cycle in a connected graph G with n vertices, then the length of the Hamiltonian path is $n-1$. In this paper, we have tried to collect conditions for the existence of Hamiltonian graphs.

2. Hamiltonicity in Random Graphs

A classic theorem by Dirac [19] implies that the local resilience of K_n with respect to the containment of a Hamilton cycle is $1/2+o(1)$. Asymptotically this remains true if K_n is replaced by a Sparser graph. For instance, it was shown by Lee and Sudakov [40] that the random graph $G(n,p)$, which is defined on n vertices with each pair of vertices forming an edge independently with probability p, satisfies the following with high probability if $p=\Omega(logn/n)$: whichever edges an adversary removes from $G(n,p)$ respecting that $1/2+o(1)$ of the incident edges remain at every vertex, the resulting graph is still Hamiltonian. Towards the advancement of random graphs, Koml*ó*s and Szemer*é*di [37] and Bollob*á*s [8] stated that a random graph $G\sim G(n,p)$ with $np-ln\,n-ln\,ln\,n\longrightarrow\infty$, is asymptotically almost surely Hamiltonian. If $np-ln\,n-ln\,ln\,n\longrightarrow-\infty$, then asymptotically almost surely $\delta(G)\leq 1$, and thus the graph G is not Hamiltonian and this is true for $\delta(G)\geq 2$. Hefetz, Krivelevich and Szab*ó* [32] proved the if $p\leq\frac{1}{2}$ is such that $\frac{p.n\,ln\,ln\,ln\,ln\,n}{ln\,n.\,ln\,ln\,ln\,n}\to\infty$, then the probabability of $G\sim G(n,p)$ having $\delta(G)<2$ is of order $\ominus(np(1-p)^n)$, and $1\leq\frac{Pr\,(G\,is\,not\,Hamiltonian)}{Pr\,(\delta\,(G)<2)}\leq exp\,(O(\frac{ln\,n\,ln\,ln\,ln\,n}{ln\,ln\,ln\,ln\,n}))$. In 2020, Yahav Alon and Michael Krivelevich [5] proved the following: Let $0\leq p=p(n)\leq 1$ and let $G\sim G(n,p)$, then

- *Pr(G is not Hamiltonian) = (1 + o(1)) Pr(*$\delta(G)<2$*)*
- *Pr(G contains no perfect matching) = (1 + o(1)) Pr(*$\delta(G)=0$*)*

For a given graph Γ_n with n verteices and m edges, Tony Johansson [36] defined the Erdös-R*é*nyi graph process with host graph Γ_n and proved that with high probability the graph $\Gamma_{n,t}$ where $t\leq m$ becomes Hamiltonian at the same moment its minimum degree reaches 2.

3. Independent Conditons

Hamiltonian closure of a graph G is a subgraph of G with vertex set V obtained by repeatedly adding edges between nonadjacent vertices $u,v\in V$ such that $d(u)+d(v)\geq n$ untill no such pair of vertices exist. A simple graph with n vertices is Hamiltonian if and only if its closure is Hamiltonian [54]. In 1884 Tait [53] conjectured that every 3-connected planar cubic graph has a Hamiltonian cycle. In 1946, Tutte [54] constructed a graph as a counterexample with 46 vertices, 69 edges and 25 faces and refused Tait's conjecture. Grinberg's theorem [31] gives the necessary condition for a planar graph to be Hamiltonian.

Xia Hong and Huihui Zhang [34] gave Hamilton sufficient condition for completely independent spanning trees. For k spanning trees $T_1,T_2,...,T_k$ of a graph G, they called these k spanning trees completely independent, if the paths from the vertex u to v of G in these k trees are pairwise openly disjoint. It has been asked whether sufficient conditions for Hamiltonian graphs are also sufficient for the existence of two completely independent spanning trees. They [34] also proved that if G is a graph with n vertices and $|N(x) \cup N(y)| \geq \frac{n}{2}, |N(x) \cup N(y)| \geq 3$ for every two nonadjacent vertices x, y of G, then it has two completely independent spanning trees. In a note, Qin, Hao, Pai, Chang [51] first attend the restriction on the number of vertices and point out that there is a flaw in their proof and gave an amendment to correct the proof.

4. Spectral Conditions

Let $A(G)$ be the adjacency matrix of the graph G. The largest eigenvalue of $A(G)$ is said to be spectral radius of G and it is denoted by $\rho(G)$. The matrix $Q(G) = D(G) + A(G)$ is the signless Laplacian matrix, where $D(G)$ is the degree diagonal matrix of G. The signless Laplacian spectral radius is denoted by $q(G)$. Sufficient conditions for a graph to be Hamiltonian and traceable in terms of spectral radius are given by Fiedler and Nikiforov [26]. The signless Laplacian spectral radius of the complement of a graph is scrutinized by Zhou [57] and provided sufficient conditions for the existence of Hamiltonian graph. Let G be a bipartite graph with X and Y be the bipartition of the vertex set V of G. Let G^* be the quasi-complement of G, where $|X| = |Y| = n \geq 2$. If $\rho(G^*) \leq \sqrt{n-1})$ then G is Hamiltonian [42]. Similarly, if $\rho(G^*) \leq \sqrt{\frac{n-2}{2}}$ then G is Hamiltonain [41]. Lu, Liu and Tian [44] proved for a bipartite graph G of size m with bipartition X and Y, where $|X| = |Y| = n \geq 2$. If $\delta \geq 1, m \geq n^2 - n + 1$, then G is Hamiltonian unless $G \cong K_{n,n-1} + e$. Bipartite graph with $\delta \geq 2$ and with m edges, where $|X| = |Y| = n \geq 4$, if $m \geq n^2 - 2n + 4$, then G is Hamiltonain unless $G \cong K_{n,n-2} + 4e$ [42].

5. Hamiltonicity of Kneser Graphs

The Kneser graph $K(n, k)$ has as vertices the k-subsets of $[n]$. Two vertices are adjacent if the k-subsets are disjoint. The Kneser graph $K(2k-1, k-1)$ is an odd graph O_k for $k \geq 2$. In 1972, Lloyd and Meredith [43] proved that the odd graph O_k is Hamiltonian for $4 \geq k \geq 7$. Mather [45] proved that the odd graph O_k is Hamiltonian for $k \geq 8$ if $k \neq 3$. Heinrich and Wallis [33] proved that $K(n, k)$ is Hamiltonian for $k \leq 8$ when $n \geq 2k + 1$. Chen [14] proved that if $n \geq 3k$ then $K(n, k)$ is Hamiltonian and gave a short proof in [16] when k divides n. Chen [15] improved his results and proved the following:

- If $n \geq L(k) = \frac{3k+1+\sqrt{5k^2-2k+1}}{2}$, then $K(n, k)$ is Hamiltonian.
- If $n \geq L(k) = \frac{3k+1+\sqrt{5k^2-2k+1}}{2}$, then $H(n, k)$ is Hamiltonian.

J. Bellmann and B. Schulke [7] proved that the Kneser graphs $K(n, k)$ are Hamiltonian for $n \geq 4k$ and bipartite Kneser graphs $H(n, k)$ is Hamiltonian for $n > 3k$.

6. k-Connected Graphs

A graph G is said to be k-connected if there does not exist a set of $k-1$ vertices whose deletion disconnect the graph G. In other words, if the vertex connectivity of a graph G is at least k, then the graph G is said to be k-connected. In this section, we accumulate some of the best known conditions for a 2-connected graph G to be Hamiltonian.

- $ab \notin E \Rightarrow |N(a) \cup N(b)| + min\ \{d(u)|u \in V\} \geq n$ [24]
- $ab \notin E \Rightarrow |N(a) \cup N(b)| \geq \frac{2n-1}{3}$ [25]
- $ab \notin E \Rightarrow |N(a) \cup N(b)| + d(a) + d(b) \geq (2n-1)$ [8]
- $ab \notin E \Rightarrow N(a) \cup N(b) + max\{d(a), d(b)\} \geq n$ [27]
- $d(u) + d(v) + d(w) \geq n + |N(u) \cap N(v) \cap N(w)|$ [27]
- $ab \notin E \Rightarrow |N(a) \cup N(b)| \geq \frac{2n-3}{3}$ [1],[6]
- $ab \notin E \Rightarrow |N(a) \cup N(b)| + min\{d(u)|u \notin \{a, b\} \cup N(a) \cup N(b)\} \geq n$ [2]

Chvátal and Erdös [18] proved for a k-connected graph G that if the independence number α if $\alpha \leq k$, then G is Hamiltonian. For a k-connected graph of order n, suppose $\exists$ some s, $1 \leq s \leq k$ such that for all independent set $S \subset V(G)$ of cardinality s, we have $d(S) > \frac{s(n-1)}{s+1}$, then G is Hamiltonian [28]. Ainouche [2] has been updated the previous results such that if $d(S) > (\frac{s(n-1)}{s+1}) - \frac{k}{s+1}\left\lfloor \frac{s}{2} \right\rfloor - \frac{j(S)}{s+1}\left\lfloor \frac{s-1}{2} \right\rfloor$, then G is Hamiltonian. Ainouche [2] has been continuously done work in this regard and gave sufficient conditions for a k-connected graph G of order n to be Hamiltonian. These sufficient conditions are :

- $d(S) > (\frac{s(n-1)}{s+1}) - \frac{k}{s+1}\left\lfloor \frac{s}{2} \right\rfloor$
- $d(S) > \frac{k}{k+1}(n - \left\lfloor \frac{k}{2} \right\rfloor - 1)$
- $d(S) > (\frac{s(n-1)}{s+1}) - \frac{\lambda(S)}{s+1}(\frac{\lfloor \frac{s}{2} \rfloor}{\lfloor \frac{s+1}{2} \rfloor}) - (\frac{j(S)}{s+1})\left\lfloor \frac{s-1}{2} \right\rfloor$
- $d(S) > (\frac{s(n-1)}{s+1}) - \frac{\lambda(S)}{s+1}(\frac{\lfloor \frac{s}{2} \rfloor}{\lfloor \frac{s+1}{2} \rfloor})$
- $d(S) > (\frac{s(n-1)}{s+1}) - j(S)\frac{\lfloor \frac{s}{2} \rfloor}{s+1}$
- $d(S) + max\{max\{d(u)|u \in S\}, min\{d(u)|u \notin S \cup N(S)\}\} \geq n$
- $d(S) + min\{d(u)|u \notin S \cup N(S)\} \geq n$

Let G be a k-connected graph of order n. Suppose there exists some s, $1 \leq s \leq k$ such that for every independent set $X \subset V$ of cardinality $s+1$, if one of the condition :

- $d(X) + \frac{\lambda(x) + i(X)\lfloor \frac{s-1}{2} \rfloor}{\lfloor \frac{s+1}{2} \rfloor} \geq n$
- $d(X) + i(X) \geq n$
- $d(X) + \frac{\lambda(X)}{\lfloor \frac{s+1}{2} \rfloor} \geq n$

is satisfied, then G is Hamiltonian [2].

In 1984 Fan [23] generalized Ore's and Dirac's conditions and proved that if a 2-connected graph of order n and $max\{d(x), d(y)\} \geq \frac{n}{2}$ for each pair of nonadjacent vertices x and y with $d(x, y) = 2$ in G, then G is Hamiltonian. In 1993, Chen [13] proved that if G is a 2-connected graph of order n, and if $max\{d(x), d(y)\} \geq \frac{n}{2}$ for each pair of nonadjacent vertices x, y with $1 \leq |N(x) \cap N(y)| \leq \alpha(G) - 1$, then G is Hamiltonian. Chen, Egawa, Liu and Saito [17] further showed that if G is a k-connected graph of order n, and if $max\{d(v) : v \in S\} \geq \frac{n}{2}$ for every independent set S of G with $|S| = k$ which has two distinct vertices $x, y \in S$ such that $d(x, y) = 2$, then G is Hamiltonian. In 2007, Zhao, Lai and Shao [56] generalize the conditions given in [13], [17], [19], [23] and proved that if G is a k-connected graph of order n and if $max\{d(v) : v \in S\} \geq \frac{n}{2}$ for every independent set S of G with $|S| = k$ which has two disjoint vertices $x, y \in S$ satisfying $1 \leq |N(x) \cap N(y)| \leq \alpha(G) - 1$, then G is Hamiltonian. In 1952, G. A. Dirac [19] proved the following theorem : let G be a finite connected graph with single edges and no separating vertices, then either G has a Hamiltonian circuit or the maximal length c of a circuit satisfies $\lfloor \frac{c}{2} \rfloor \geq \rho_0$, where ρ_0 is the smallest degree of any vertex. This theorem has also been proved and extended by Erdös and Gallai [20] and by Ore [48]. In 1967, Ore [49] improved Dirac's theorem and gave a result, when G has no Hamiltonian circuits, then $l \geq \rho_0 + \rho_1 + 1$ except for the two types of graphs : (i) star graphs $S_r(H)$ where H has $\rho + 1$ vertices and $r \geq \rho_0 + 2$ (ii) one-vertex composition of $k \geq 3$ complete graphs on $\rho_0 + 1$ vertices. Here, we are discussing about a very important topic in graph theory which is platonic graph. In three-dimensional space, a solid constructed by polygonal faces with the same number of faces meeting at each vertex which are congruent (identical in shape and size) and regular (all angles equal and all sides equal) is called platonic solid. There are five solids which meet these criteria: tetrahedron, cube, octahedron, dodecahedron and icosahedron. In 2004, Brain Hopkins [35] developed a combinatorial method to show that the dodecahedron graph has, upto rotation and refection, a unique Hamiltonian cycle. They called platonic graphs with this property, topologically uniquely Hamiltonian. They used the same method to demonstrate topologically distinct Hamiltonian cycles on the icosahedron graph and to show that a regular graph embeddable on the 2-holed torus is topologically uniquely Hamiltonian.

7. Graphs with Moderate Degree

Gordon [29] proved that there are some non-Hamiltonian regular graphs with $2n$ vertices. A 2-connected $2n$ vertices regular graph of degree $n - 2$ and $n \geq 6$ is Hamiltonian [22] and extended this result for regular graphs of degree $n - k$, $k \geq 3$ [21]. Erdös and Hobbs [21] proved that if k is an integer not less than 3, and if G is a 2-connected graph with $2n - a$ vertices, $a \in \{0, 1\}$, which is regular of degree $n - k$, then G is Hamiltonian if $a = 0$ and $n \geq k^2 + k + 1$ or if $a = 1$ and $n \geq 2k^2 - 3k + 3$. Let P be the longest cycle in G and $R = V(G) - V(P)$. Let v and w be in R. Then v is not adjacent to any vertex in $A_v \cup B_v$, A_v and B_v are independent sets of vertices, and w is joined to at most one vertex of A_v and to at most one vertex in B_v. Erdös and Hobbs [21] also given that if $n \geq 3k + 2 - a$, then R is independent and if $n \geq k^2 + k + 1$, then $r + s \leq k$. Rahman, Kaykobad and Firoz [52] proved that let $P =< x_1, x_2, ..., x_p >$ be a longest path of G such that $p \geq 4$. If $d_{x_1} + d_{x_p} \geq p - 1 + l, l \geq 1$, then there exists at least l crossover edges. They also proved for all pairs of nonadjacent vertices u, v one has $d_u + d_v \geq n - 2$, then G has a Hamiltonian path. Some of the other best known conditions are also given in [4], [7], [8], [11], [12], [29], [30], [38], [39], [46], [50], [55].

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