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FUZZY BASED COMBINATORIAL OPTIMIZATION OF TRAVELLING SALESMAN PROBLEM USING TWO OPTIMAL METHOD

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Abstract. In Operations Research field, Travelling Salesman problem (TSP) is one of an important technique and considered as one of the classical NP-complete graph search problems. Traditional TSP solutions are depending on the premise that transit periods within nodes determines their relationship by distance only. In fact, however, this is not the case because road and traffic network circumstances influence the time it takes to travel within nodes. Fuzzy based TSP solution has been used because distance and intensity of road traffic circumstances are inherently ambiguous. The result obtained using conventional type solution method for the shortest path travel is 192 minutes as an average time, while the solution using fuzzy based TSP is 177 minutes and using two optimal methods, we get 167 minutes. The potential of the proposed method based on Fuzzy TSP is better than the conventional type of solution method. The present work can be further extended for the large networks in business and industries.

1. Introduction

The travelling salesman problem (TSP) is one of the most important concerns in combinatorial optimization problems employed in many engineering fields, and it has piqued the interest of many scientists and academics. [18, 20]. Travelling Salesman Problem is one of the problems which is consider as like a puzzle. If a person (like delivery boy) wants to travel many cities or places allotted to him and salesman knows the distance, cost, time. After-all salesman must return from the last city to his original city. In this a traveler must visit every city at least once and return to its original city to visit every city in which minimum distance (or cost, or time). This is not only the case of shortest route although it is case of one city to other city's road conditions and road traffic[6, 9].

Overall, the meaning of travelling salesman problem is this to find out the minimum (least) cost or time or distance to visiting all the cities at least once in which starting and ending point will be same [8]. The Travelling Salesman Problems (TSP) mainly consider for the shortest route, it means if a person wants to travel one city to another city. This technique provides a better and shortest path (minimum time, minimum cost) for travelling. A classic common combinatorial optimization issue and an NP-hard problem is the travelling salesman problem.

Key words and phrases. Optimization, TSP, Type-1 fuzzy set, Type-2 fuzzy set, Two optimal, Networks flow.

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Transportation issues were expanded to include several objectives (MOTP). Finding the perfect answer for every problem at once is not possible [7]. Travelling salesman problem are solved with different techniques and different rules by some researchers like a modified metaheuristic algorithm, artificial cooperative search algorithm, discrete cuckoo search algorithm, genetic algorithms, and time dependent intuitionistic valued also [2, 10, 14, 15, 20]. But sometimes during travelling road condition also matters like traffic, determining road. In this situation fuzzy based solution works well. Fuzzy set theory was introduced by Lotfi Asker Zadeh in 1965. L.A Zadeh and Bellman introduced fuzzy dynamic programming. In the Time Dependent Traveling Salesman Problem, the "distances" (costs) between nodes vary in time, they are considered longer during the rush hour period or in the traffic jam region[2]. Engineers and scientists have long used fuzzy sets to solve problems involving uncertainty. It has also been used to solve the problem of mathematical programming. TSP are a type of optimization problem in which a salesman travels from a starting city D_1, like as the city shown in Figure 1, to subsequent cities then returns to the starting city from the last city D_5. Overall the target of the salesman finds the shortest route to minimize the distance travelled by the salesman. The model for this research study included the road and traffic circumstances. The model for this research study considers the road and traffic circumstances. The TSP is shown mathematical form in equation (I).

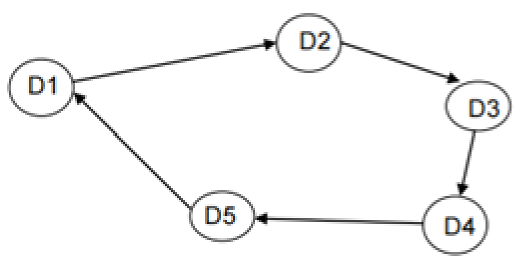

FIGURE 1. Travelling salesman problem.

$$Minimize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{n} d_{ij}x_{ijt} \tag{1.1}$$

n =total number of stops.
i, j, t = indices of nodes that accept integer values from 1 to n.
t = the time.
x_{ijt} = 1. (If the edge of the network from$i$to j, otherwise 0).
d_{ij} = the total distance travelled by salesman (or cost) from i to j .

2. Network Optimization

Travelling salesman problem can be readily transformed in the networks problem. In this case study a travelling salesman travels from one city to other city then he chooses an optimal route and during whole journey creates some networks. Network is a graphical model in which some commodity is transported from one place to another place. In Network Optimization the tool and the applied technique that help to improve, maintain, maximize network performance Network performance monitoring includes measuring traffic, jittery bandwidth latency caused by the likes of sufficient infrastructure. Networks optimization generally initiates optimizing and leveraging technologies for existing network system. Network optimization used for shortest path, maximizing flow, multi-commodity flow, network design, minimum spanning trees, travelling salesman problem also.

3. Hamiltonian Circuit and Paths

Travelling salesman problem can be formulated and solve using graph. In some cases, when travelling salesman required to visit each city once and once only then created a circuit. Such a circuit is known as Hamiltonian circuit. Irish mathematician Sri William Hamilton who introduced Hamilton graph. In which he introduced a puzzle, named as Icosran game. In Icosran game has twenty vertices and make a decahedron with name of every city and target is to visit every city at least once and necessary to reach its starting city to ending city makes a graph and this graph is known as Hamiltonian Graph. In tour, starting and ending points are same then it's making a circuit that is Hamiltonian circuit. Hamiltonian Graph needs Hamiltonian circuit in which same number of the vertices and edges. For travelling salesman problems in the graph vertices likes cites to be visited by salesman. And weight of the edges between the vertices i and j denote the distance between the places$i$ and$j$.

3.1. Rule.

(i) Let G is the graph and number of vertices is n then Hamiltonian path contains only $n-1$ edges and Hamiltonian Cycle must contain only 'n' edges. choose any vertex v check all the vertices adjacent to v .Find the vertices closet to v and take a path to joining to v and u.

(ii) Choose a vertex to the last vertex which is not chosen before by checking all the vertices adjacent to it as in above (i).

(iii) If there is no vertex left, then join the vertex v and the last one vertex choose. Otherwise follow back (ii).

4. Fuzzy Sets

Fuzzy sets are generally called Type-1 fuzzy set. Fuzzy sets that generalize the classical, the degree of membership in crisp sets defines them. The classical set is the special case of membership function of fuzzy sets where character takes only 0 and 1 [13].Fuzzy concept is very useful to solve real life problem.The fuzzy study emphasizes the integration of sustainability into industrial practices, highlighting the significance of green supply chain (GSC) management through supplier selection (SS) and the application of multi-criteria decision-making (MCDM) techniques, particularly combining analytical hierarchy process (AHP) and fuzzy TOPSIS, to optimize electric vehicle selection, multi-objective in fuzzy environments[2-3, 17]. For example, let us consider that crisp set of big doors, it is possible to be a part of it or not. However, in a type-1 fuzzy set of circumstances, big doors something difference, the membership is defined by values in which complete membership gives 1 and 0 for non-membership. Hence, big, and huge may belong to this to 1 while little door belongs to zero. For a crisp set A, huge door that represents an element is a member in the universe or not. It's shown logically as:

$$Y_A(y) = \begin{cases} 1, & y \in A \\ 0, & y \notin A \end{cases} \tag{4.1}$$

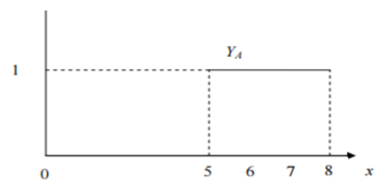

FIGURE 2. Crisp set representation.

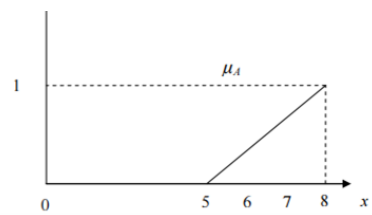

FIGURE 3. Fuzzy set representation

Let us assume that door's height maximum 8 feet. Now we can show it in the crisp set and type-1 fuzzy set. Look in the figure:2 where H represents a set of door's heights are thought to be tall. Because tallness is a fuzzy attribute, H has no specific membership function. As we have considered the maximum height for doors is eight feet and any doors with the same height or less than five feet is not to be considered in tallness as shown in the figure:3. $\mu_H$denotes the membership function, is.

$$\mu_H(8) = 1, \mu_H(5) = 0 \tag{4.2}$$

Let Y be a classical, crisp set of the object called the Universe. When the discourse universe,Y. Type-1 fuzzy set A: fuzzy sets is finite and discrete then.

$$A = \mu_A(y_1)y_1 + \mu_A(y_2)y_2 + .. = \left\{ \sum_i \frac{\mu_A(y_i)}{y_i} \right\} \tag{4.3}$$

Type-1 fuzzy set A: fuzzy set is finite and continuous then

$$A = \left\{ \int \frac{\mu_A(y)}{y} \right\} \tag{4.4}$$

4.1. Type -2 fuzzy Set. The extension of type-1 fuzzy sets is type-2 fuzzy sets and type-2 fuzzy models makes it easy to solve the problem of uncertainty [11, 12, 16]. The fuzzy membership function describes a type-2 fuzzy set. Type-2 fuzzy set membership grade is a fuzzy set in [0, 1], whereas Type-1 fuzzy set membership grade is a crisp value in [0, 1]. Type-2 fuzzy logic gives the best prediction accuracy according to the experimental data [1, 5]. These sets are used whenever the situations are uncertainty or fuzzy in which there is a problem determining the membership grade as a crisp value in the range [0, 1]. Thus, in real-world problems, there are two types of fuzzy sets: type 1 and type 2 as the first and second order approximations to the uncertainty. A Type-2 fuzzy set $\widetilde{\widetilde{A}}$ on the discourse universe Y is defined by its membership function $\mu_{\widetilde{\widetilde{A}}}(y,u)$, and characterized as follows:

$$\widetilde{\widetilde{A}} = \{(y,u), \quad \mu_{\widetilde{\widetilde{A}}} = y, u \mid \text{ for all } y \in Y,\ u \in J_y \subseteq [0,1]\}$$

where $0 \leq \mu_{\widetilde{\widetilde{A}}}(y,u) \leq 1$, the subinterval J_y is the primary membership of y, and $\mu_{\widetilde{\widetilde{A}}}(y,u)$ is the role of secondary membership. Thus, Type-2 fuzzy set $\widetilde{\widetilde{A}}$ is described as:

$$\widetilde{\widetilde{A}} = \int_{y\in Y}\int_{u\in J_y} \mu_{\widetilde{\widetilde{A}}}(y,u)/(y,u)\,dy\,du, \quad J_y \subseteq [0,1]$$

where $\int\int$ represents complete allowable y and u. $\sum$ has been used instead of $\int\int$ for discrete universal set.

4.1.1. *Type-2 triangular fuzzy number.* Type-2 triangular fuzzy number is defined as
$\widetilde{\widetilde{A}} = \left\{\left(x, {\mu_A}^L(x), {\mu_A}^M(x), {\mu_A}^N(x)\right)\right\}$ and ${\mu_A}^L(x) \leq {\mu_A}^M(x) \leq {\mu_A}^N(x)$ for all $x \in R$.

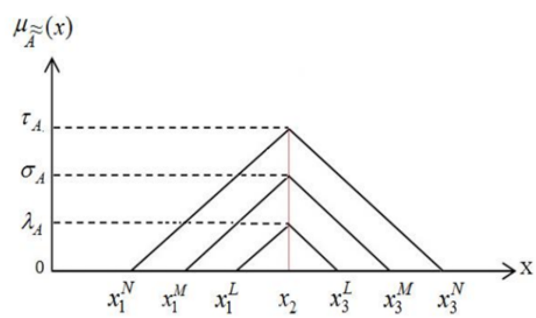

FIGURE 4. Type-2 triangular fuzzy number.

5. Methodology

We solve the TSP by two optimal methods first, in this method a tour is designed in another arbitrary way after those two links (i, j) broken and the path which are left should joined up so that to construct new tour. If the length L is less than the length of the original tour (previous tour). Then the new tour will be retained and repeated the starting process shown in figure 5.

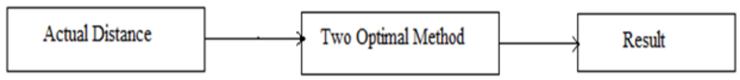

FIGURE 5. Interface of the method

Algorithm for solution

i Create an initial tour.
ii Set $i = 1$, select a starting point s.
iii Now choose the smallest number t.
iv Select t from unvisited nodes that is least. Repeat until the end of the tour.
v Add all the s in tours so that the total length is L.
vi Now for the second tour set $j = i + 2$, so that $d(s, t) \leq d(s, j)\ \forall\ j \neq t$.
vii Let the tour be $x_1, x_2, x_3, \ldots, x_n, x_j, x_{j-1}, \ldots, x_{i+1}, x_{j+1}, \ldots, x_1$. Created by exchanging the links (x_i, x_{i+1}) and (x_j, x_{j+1}). If this length is less than L, then make the next tour and follow steps (iv) and (v).
viii Select $j = j + 1$ if $j \leq n$ and repeat step (vi). Otherwise, set $i = i + 1$ and if $i \leq n - 2$, follow step (v).

Another way to solve TSP in type-1 fuzzy, in this case the travelling salesman problem (TSP) consists of five cities as shown in figure 1. The TSP started to move from the city named D1 and delivers merchandise to customers in the following four cities. The optimal route was determined using the conventional TSP optimization technique, which ignored the road and traffic conditions. The type-1 fuzzy-TSP algorithm was developed in considering road conditions and traffic to establish a best new path for the salesman in the context of type-2 fuzzy modelling. This idea came about because of fuzzy inference. Following the computations, the best routes were selected using type-2 fuzzy and two optimal methods. The salesmen were put to the test to find out the time taken along various routes. The inference of type-1 fuzzy system consists of the actual distance and road condition shown in figure-6.

6. Problem Description and solution

Fuzzy transportation problem is solved by the proposed method called two optimal methods where the data for the problem is taken from the problem described in the paper "Fuzzy Based Solution to the Travelling Salesman Problem: A Case Study". There are five cities namely D1, D2, D3, D4, and D5 that are mentioned with their distance matrix given in table 1. It is assumed that city 1 is source

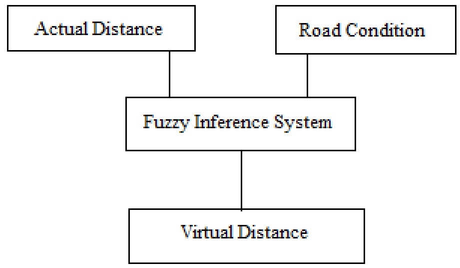

FIGURE 6. Fuzzy inference system

node where salesman move to other four city and back to the source city. The objective of this study is to minimize the total distance (cost, time) without violating the traditional approach. Therefore, we use two optimal methods to solve the travelling salesman problem in fuzzy environment and compare with result with traditional approach.

TABLE 1. Actual distance Matrix

	D1	D2	D3	D4	D5
D1	0	31	40	24	27
D2	31	0	69	47	42
D3	40	69	0	27	43
D4	24	47	27	0	43
D5	27	42	43	43	0

Shown in Figure 7. Solution using two optimal methods as:

- Step (i): $i = 1$, $s = 1$.
- Step (ii): $t = 4$.
- Step (iii): $D1 \rightarrow D4 \rightarrow D3 \rightarrow D5 \rightarrow D2 \rightarrow D1$.
- Step (iv): $27 + 27 + 43 + 42 + 31 = 167$.

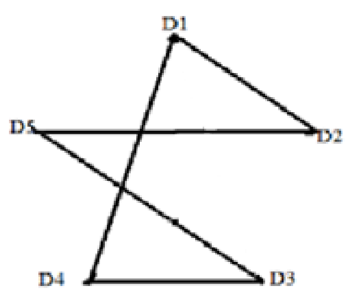

FIGURE 7. First network tour

Shown in Figure 8.
Step (v): $i = 1,\ j = 3$.
Step (vi): $D1 \to D5 \to D2 \to D4 \to D3 \to D1$
$L = 183$.

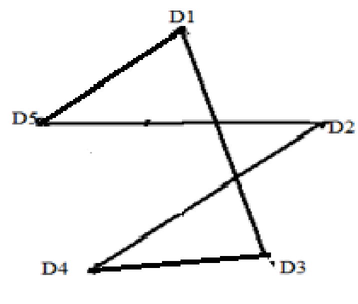

FIGURE 8. Second network tour

Shown in Figure 9.
Step (vii): $j = 4$.
Step (vi): $D_1 \to D_2 \to D_5 \to D_3 \to D_4 \to D_1$
$L = 167$

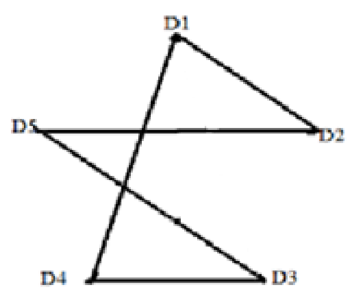

FIGURE 9. Third network tour.

Shown in Figure 10.
Step (vii): $j = 5$.
Step (vi): $D_1 \to D_3 \to D_4 \to D_5 \to D_2 \to D_1$
$L = 183$.

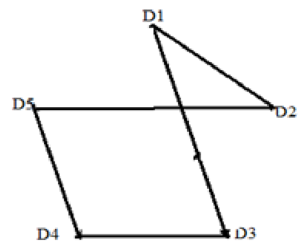

FIGURE 10. Fourth network tour.

Shown in Figure 11.
Step (vii): $j = 6$, but $j \leq n$, then Step (vi); otherwise $i = i+1$, so $i = 2$, $j = i+2$, so $j = 4$;
Step (vi): $D_1 \to D_4 \to D_5 \to D_2 \to D_3 \to D_1$
$L = 218$.

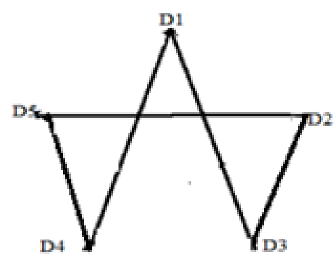

FIGURE 11. Fifth network tour.

Shown in Figure figure12.
Step(vii) $j = 5$;
Step(vi) $D1 \to D4 \to D3 \to D5 \to D2 \to D1$

$$L = 167.$$

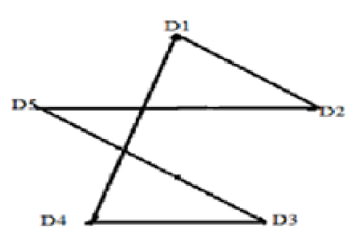

FIGURE 12. Sixth network tour.

Shown in Figure figure13.
Step(vii) $j = 6, \quad \text{so} \quad i = 3;$
Now $j = 5;$
$D1 \rightarrow D4 \rightarrow D3 \rightarrow D5 \rightarrow D2 \rightarrow D1$
$L = 167.$

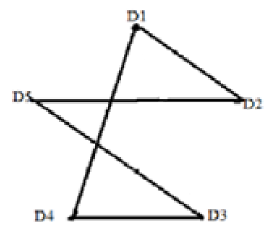

FIGURE 13. Seventh network tour.

Now when we solve TSP table-1 in fuzzy then we start again from beginning, the travelling salesman travels five cities in this case as mentioned in table 2, so now the road and traffic conditions were mentioned like $Level_1(L1), Level_2(L2), Level_3(L3), Level_4(L4) and Level_5(L5)$. Here, very small, small, medium, long, very long these are the routes distances were shown. The linguistic variables were used to generate membership functions. These are shown in figures-14 and figure-15 below.

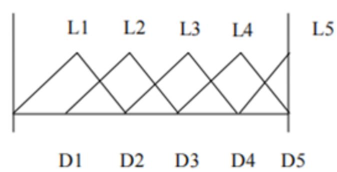

FIGURE 14. Membership function for road and traffic conditions.

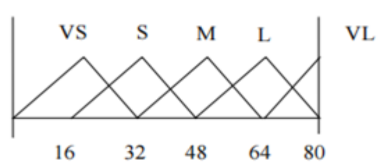

FIGURE 15. Membership function for actual distances

For salesman rules are created According to the road conditions and traffic condition during travel. For examples if the conditions of road are very good, then travel time could be to find out by the equation:

$$time = \frac{distance}{speed}$$

One thing to note that time travel could be double when road condition is not good so salesman would have more travel. Then extra distance travel by salesman is known as virtual distance in table 3.

Table 2. Fuzzy Matrix

	D1	D2	D3	D4	D5
D1	-	Level_1	Level_2	Level_2	Level_2
D2	Level_1	-	Level_3	Level_4	Level_3
D3	Level_2	Level_3	-	Level_2	Level_1
D4	Level_1	Level_4	Level_2	-	Level_2
D5	Level_2	Level_3	Level_1	Level_2	-

Table 3. Virtual Distance Matrix

	D1	**D2**	**D3**	**D4**	**D5**
D1	-	60.6	72.0	48.0	51.2
D2	60.6	-	102.0	47.0	57.6
D3	70.0	102.0	-	73.6	84.2
D4	48.0	47.0	73.6	-	74.5
D5	52.2	57.6	84.2	74.5	-

Table 4. Route and Cost

Description	**Route**	**Cost**
Normal TSP	$D1 \rightarrow D4 \rightarrow D3 \rightarrow D5 \rightarrow D2 \rightarrow D1$	322.00
Fuzzy TSP	$D1 \rightarrow D5 \rightarrow D2 \rightarrow D4 \rightarrow D3 \rightarrow D1$	302.00
Current Path	$D1 \rightarrow D4 \rightarrow D2 \rightarrow D5 \rightarrow D3 \rightarrow D1$	308.00

7. Results

After applying this method, I have taken the problem and given the result in table 5. The result is optimum as normal TSP, fuzzy TSP, current path, two optimal methods. Thus, two optimal methods are minimum to the other result like normal TSP, fuzzy TSP, current path.

Table 5. Route and time of TSP

Route	**First time**	**Second time**	**Third time**	**Average**
Optimal route by TSP	185	189	183	185
Optimal route by organization	194	190	192	192
Optimal route by fuzzy	181	175	177	177
Optimal route by two optimal methods	167	167	167	167

8. Conclusions

This paper highlights the importance of two optimal methods to optimize the combinatorial network problem. The result obtained using the first three conventional is higher than the result obtained by using that two optimal method for minimization of an objective. This shows that the proposed method under the problem consideration is optimal. The result of this study shows a novel way to solve the problem both in fuzzy and non-fuzzy environments. The solution methodology and concept can be further extended to businesses and industries.

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