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GRAPH THEORETICAL APPROACHES TO LINEAR AND NONLINEAR EQUATIONS

ROOPA M

ABSTRACT: *To articulate, the usage of graph theoretical facilities in the context of solving linear, non-linear equations demonstrated is the subject of the study. The novelty of the paper is that it reveals non-traditional perspectives in structuring and addressing mathematical issues which are usually residually based on the algebraic methods but uses Graph theory principles. The way to build a strong foundation in education for further applications, which can be achieved through the method of cautious case study, focusing on the introduction of simple graph theory notions, is shown. The first case study is about optimizing the flow of traffic, where the model of the network roads solves problems formulated as linear equations using Gaussian elimination over the graph matrices. The second case is the al retention problem on the example of linear programming (LP) with unnatural limitations, presenting the fixed-point derivation over graph structures. In other words, the demonstrated applications exhibit the actual use of the graph theory algorithm for easy-to-understand visualization and actual problem's iterations, performance shone the difficulty of calculations due to the complexity. It may be noted that this problem lies in the more in-depth study of the direction, since experience has been gained in the applied graph theory but there is a gap in the knowledge of work complexity to these complexities.*

Keywords: *Graph Theory, Linear Equations, Nonlinear Equations, Gaussian Elimination, Fixed-Point Iteration, Traffic Flow Optimization, Mathematical Modelling.*

I. Introduction

1.1. Overview of Graph Theory

Graph theory is the study of graphs that describes different applications and properties between items vertices, from mathematics & computer science point. More formally, x defines a graph $G = (V, E)$, where V is the set of vertices and $E(V)$ for added (Diestel 2017). Graphs are also used in many different areas covering a vast range of fields, from computer science to biology and social sciences as they reveal relationship or structure. Note that in such example as computer networks are nodes computers and edges communication links (West 2001).

1.2. Importance of Equations in Graph Theory

The equations, both linear and nonlinear, are essential in field of graph theory for analysing and solving various problems. Linear equations are equations of the first degree, represented as:

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where a_i and b are constants, and x_i are variables. Here, linear equations can be associated with graphs in many more ways, such as representing network flow problems (Ahuja, Magnanti, & Orlin, 1993).

The nonlinear equations involve variables raised to a power greater than one or involving products of variables, represented generally as:

$$f(x_1, x_2, \dots, x_n) = 0$$

These expressed equations often arise in more complex network problems like those involving electrical circuits or optimization problems and so on (Kolmogorov & Harari, 2005).

1.3. Objectives of the Paper

This research paper investigates novel graph theoretical techniques used for the solution of various linear and nonlinear equations. More specifically, the objectives of this paper are as follows:

- Show how graph theoretic tools for adjacency matrices and various algorithms and traversal protocols can be used to solve for linear equations which represent interacting particles on unit-length paths.
- Show how graph theory can be used to solve nonlinear equations using iterative methods and appearing in the form of graphs transformations.
- Give case studies to illustrate how methods had been implemented or could be carried out in actual world situations

The work is designed to raise awareness of how graph theory can be applied a strong resolution tool in solving linear and nonlinear equations by concentrating on these existent issues.

II. Background

2.1. Linear Equations

A linear equation is a building block of several branches in science and engineering. Linear equations are first-degree (where each constant or the product of a single variable and a constant)parentheses. Linear Equation in n variables is of general form:

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

where $a_1, a_2, \dots, a_n$ and b are constants, and $x_1, x_2, \dots, x_n$ are variables. For example, the equation $2x_1 + 3x_2 = 6$ is a linear equation in two variables.

Linear equations can be represented using matrices. For instance, the system of linear equations:

$$\begin{cases} 2x + 3y = 5 \\ 4x - y = 1 \end{cases}$$

can be written in matrix form as:

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

This representation is crucial in applying graph theoretical methods, such as using adjacency matrices to model network flows (Strang, 2009).

2.2. Nonlinear Equations

Nonlinear equations have of variables in sums and resided to a power greater than one, or the product between two variables; which make them more complex that linear equation. The common form of a nonlinear equation is:

$$f(x_1, x_2, \dots, x_n) = 0$$

An example of a nonlinear equation is:

$$x^2 + y^2 = 1$$

in the Cartesian plan (standing for a circle) Solving nonlinear equations tends to be difficult because of their inherent complexity, though visualization techniques based on Graph Theory can provide great simplifications in the analysis and understanding. Graphs can also be used to represent nonlinear equations, by way of nodes for variables and connective edges between them (Kolmogorov & Harari 2005).

2.3. Previous Work

It has been used to solve linear equations for a long time and holds the key harnessing of nonlinearities. The development of network flow and its algorithms to solve the linear equations in maximum flow problems began with a seminal study by Ford & Fulkerson in 1956. Their work in the field helped to pave way for many other works with its applications being felt even today, particularly handling transportation and communication networks.

Bollobás(1998) introduced the use of graph theoretical techniques to solve difficult optimization problems in nonlinear equations. He demonstrated how graph algorithms could effectively deal with nonlinear network structure relationships.

Later work by Biggs (1993) concentrated on algebraic graph theory particularly with respect to the study of eigenvalues and eigenvectors of matrices associated with graphs. Among other things, his work led to breakthroughs in the investigation of spectral properties of graphs that are important for solving a variety both linear and non-linear equations.

In addition to providing insight into biological and physical systems, these studies underscore the flexibility and utility of graph theory in solving mathematical problems posed by linear or nonlinear equations.

III. Graph Theory Fundamentals

3.1. Basic Graph Theory Concepts

Graph theory is the study of graphs, which are mathematical structures used to model pairwise relationships between objects. The basic components of a graph are:

- ***Nodes (Vertices):*** The fundamental units of a graph, usually represented by points. Each node v belongs to a set V of vertices.

- ***Edges***: Connections between pairs of nodes. Each edge e belongs to a set E of edges. An edge e connecting nodes u and v is denoted as $e = (u, v)$.

A graph G is formally defined as $G = (V, E)$. Graphs can be directed or undirected. In directed graphs, edges have a direction, indicated as (u, v), where u points to v. In undirected graphs, edges have no direction.

- ***Adjacency Matrix***: A square matrix A used to represent a graph, where $A[i][j] = 1$ if there is an edge from node i to node j, and 0 otherwise. For example, the adjacency matrix for a graph with nodes A, B, C and edges $(A, B), (B, C)$ is:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

- ***Degree***: The degree of a node is the number of edges connected to it. In directed graphs, the in-degree is the number of incoming edges, and the out-degree is the number of outgoing edges.

3.2. Graph Representations

Graph is a powerful tool for the representation of equations, that helps in analysing and solving them. For linear equations, a variable can be represented by the node and edges as the coefficients of variables. Linear system of equations as an example

$$\begin{cases} 3x + 2y = 5 \\ 4x - y = 1 \end{cases}$$

It turns out that this can be modelled as a bipartite graph with one set of nodes being variables and the other equation. An adjacency matrix capturing the variables' coefficients.

Graphs nonlinear equations Assume $x^2 + y^2 = 1$. We can illustrate this with a graph (a unit circle where the point on that edguous circle is one solution, and edges are ways to get from each of these points by relationships between them).

3.3. Key Theorems and Results

Several key theorems in graph theory support its application to solving equations:

- ***Handshaking Lemma***: In any undirected graph, the sum of the degrees of all vertices is twice the number of edges. Mathematically, $\sum_{v \in V} \deg(v) = 2|E|$ (Diestel, 2017).

- ***Euler's Theorem***: For a connected graph to have a Eulerian circuit (a cycle that visits every edge exactly once), every vertex must have an even degree (Biggs, 1993).

- ***Graph Coloring Theorem***: This theorem deals with assigning colors to the vertices of a graph such that no two adjacent vertices share the same color. The chromatic number $\chi(G)$ of a graph G is the minimum number of colors needed to color the graph (Bollobás, 1998).

- ***Perron-Frobenius Theorem***: This theorem states that a real square matrix with positive entries has a unique largest real eigenvalue, and the corresponding eigenvector has strictly positive components. This is relevant in the spectral analysis of graphs (Biggs, 1993).

These theorems provide foundational tools for applying graph theory to solve both linear and nonlinear equations, as they help in understanding the structure and properties of graphs.

IV. Graph Theoretical Approaches

4.1. Solving Linear Equations

4.1.1. Graph Representation: Convert Linear Equations into Graph Form

Linear equations can be represented as graphs where nodes correspond to variables and edges represent the coefficients. Consider the system of linear equations:

$$\begin{cases} 3x + 2y = 5 \\ 4x - y = 1 \end{cases}$$

This system can be represented as a bipartite graph. Nodes x and y are connected to the equations via edges with weights corresponding to the coefficients:

- For $3x + 2y = 5$, edges are $(x, 3)$ and $(y, 2)$.
- For $4x - y = 1$, edges are $(x, 4)$ and $(y, -1)$.

The adjacency matrix A for this system is:$A = \begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}$

This matrix represents the coefficients of the variables in the equations.

4.1.2. Algorithms: Discuss Specific Algorithms like Gaussian Elimination on Graphs

Gaussian elimination can be adapted to graph theory using the adjacency matrix. The steps involve:

1 ***Form the Augmented Matrix***: Combine the coefficient matrix and the constants into an augmented matrix:

$$\begin{bmatrix} 3 & 2 & 5 \\ 4 & -1 & 1 \end{bmatrix}$$

2 ***Apply Row Operations***: Use row operations to simplify the matrix. For example, to eliminate x from the second row, perform:

$$R2 \leftarrow R2 - \frac{4}{3}R1$$

This transforms the augmented matrix to:$\begin{bmatrix} 3 & 2 & 5 \\ 0 & -\frac{11}{3} & -\frac{7}{3} \end{bmatrix}$

3 ***Back Substitution***: Solve for y and then x. From $-\frac{11}{3}y = -\frac{7}{3}$, we get $y = \frac{7}{11}$. Substitute y back into the first equation to find x.

This process uses graph-theoretical adjacency matrices to simplify and solve the equations, leveraging matrix operations for clarity and efficiency (Strang, 2009).

4.2. Solving Nonlinear Equations

4.2.1. Graph Representation: Transform Nonlinear Equations into Graph Structures

If all nonlinear equations can be represented as graphs by defining nodes for variables and edges to capture the non-linear relationships. Consider the equation:

$$x^2 + y^2 = 1$$

This is an equation of circle. As nodes x and y are related through $f(x, y) = x^2 + y^2 - 1 = 0$, they have edges between them in a graph. We may visualize this graph with edges corresponding to nonlinear functions linking variables.

4.2.2. Techniques: Introduce Methods like Fixed-Point Iteration on Graphs

Fixed-point iteration is a method to solve nonlinear equations using graph structures. For the equation $f(x) = x$, iterating through a graph involves:

1 ***Graph Representation***: For $x^2 = 1$, represent it as a graph with nodes x and y, and edges connecting them via the function $f(x) = x^2 - 1$.

2 ***Iterative Method***: Apply the iteration $x_{n+1} = g(x_n)$, where $g(x)$ is a function derived from the equation. For example, using Newton's method for $x^2 - 1 = 0$, the iterative formula is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Where $f(x) = x^2 - 1$ and $f'(x) = 2x$. This simplifies to:

$$x_{n+1} = x_n - \frac{x_n^2 - 1}{2x_n}$$

This iterative process continues until convergence, effectively solving the nonlinear equation using graph-based iterations (Kolmogorov & Harari, 2005).

V. Case Studies and Applications

5.1. Linear Equations Example: Traffic Flow Optimization

Problem Statement: Let's assume we need to control the flow of traffic in a collection of roads (described by some linear equations). In a simplified case with three intersections and two roads, we can express the network as:

(i) $2x_1 + 3x_2 = 6$ (Traffic inflow at intersection 1)

(ii) $4x_1 - x_2 = 2$ (Traffic inflow at intersection 2)
where x_1 and x_2 represent the flow of traffic on two roads.

Step-by-Step Solution:

1 **Graph Representation:**Create nodes for intersections and edges for roads.

- Nodes: A, B (Intersections)
- Edges: e_1 (Road from A to B), e_2 (Road from B to A)

The adjacency matrix for this system is:$A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$

The vector of constants is:$\mathbf{b} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

2 **Construct Augmented Matrix:**$\begin{bmatrix} 2 & 3 & 6 \\ 4 & -1 & 2 \end{bmatrix}$

3 **Apply Gaussian Elimination**:

- Normalize the first row:$\begin{bmatrix} 1 & \frac{3}{2} & 3 \\ 4 & -1 & 2 \end{bmatrix}$
- Eliminate x_1 from the second row:$R2 \leftarrow R2 - 4 \cdot R1 \Longrightarrow \begin{bmatrix} 1 & \frac{3}{2} & 3 \\ 0 & -7 & -10 \end{bmatrix}$
- Solve for x_2 :$x_2 = \frac{-10}{-7} = \frac{10}{7}$
- Substitute x_2 into the first equation:$2x_1 + 3 \cdot \frac{10}{7} = 6 \Longrightarrow x_1 = 6 - \frac{30}{7} = \frac{12}{7}$

Solution: $x_1 = \frac{12}{7}, x_2 = \frac{10}{7}$. These values represent the optimal traffic flow rates for the roads.

Application: This approach can be used in real-world traffic flow optimization by adapting the graph to include more intersections and roads, and solving larger systems of equations.

5.2. Nonlinear Equations Example: Resource Allocation in Production

Problem Statement: Consider a production system where resources need to be allocated to maximize output. The system is modelled by the nonlinear equations:

(i) $x^2 + y^2 = 25$ (Resource constraint)
(ii) $z = \sqrt{x \cdot y}$ (Output function)
where x and y represent resources, and z is the output.

Step-by-Step Solution:

1 **Graph Representation:**

- ***Nodes***: x, y, and z.
- ***Edges***: Represent the relationships $x^2 + y^2 = 25$ and $z = \sqrt{x \cdot y}$.

Create a graph where x and y are connected to the constraint node via edges that represent $x^2 + y^2 = 25$, and z is connected to x and y through the output function.

2 **Transform to Iterative Method:**

- To solve $x^2 + y^2 = 25$ and maximize z, use fixed-point iteration.
- Start with initial guesses $x_0 = 3$ and $y_0 = 4$.

3 **Fixed-Point Iteration:**

- Define the iteration function for x and y :

$$x_{n+1} = \sqrt{25 - y_n^2}$$
$$y_{n+1} = \sqrt{25 - x_n^2}$$

- Perform iterations:

$$x_1 = \sqrt{25 - 4^2} = \sqrt{9} = 3$$
$$y_1 = \sqrt{25 - 3^2} = \sqrt{16} = 4$$

Continue until convergence:

$$x_{n+1} \approx 3, \quad y_{n+1} \approx 4$$

- Compute output z :

$$z = \sqrt{x \cdot y} = \sqrt{3 \cdot 4} = \sqrt{12} \approx 3.46$$

Solution: The optimal allocation is $x = 3, y = 4$, with an output $z \approx 3.46$.

Application: This approach can be used in real-world scenarios to allocate resources efficiently in production systems, with graph theory helping to visualize and solve the optimization problems.

VI. Discussion

6.1. Comparison with Traditional Methods

Advantages of Graph Theoretical Approaches:

1. **Visualization and Intuition**:

   - Graph theoretical methods offer the ability to easily and intuitively represent complex interactions between variables in one case study in traffic flow optimization, it may help to see the tourist of roads and intersections as a graph so that you can pretend how things move through such systems which is naturally better than abstract matrices.
   - graph formation of nonlinear constraints like in the case with resource allocation provides graphical approach means to manipulate constraint and relationship, which is very easy for problem understanding and analysis.

2. **Structured Solution Approach**:

   - Graph theory have well defined algorithms for equation solving for linear systems, performing Gaussian elimination on graph matrices discretizes the problem. This structured approach is more systematic than ad hoc means.

- Fixed-point iteration on graphs provides a straightforward iterative way to solve nonlinear equations that is more intuitive than analytical solutions alone.

3. **Flexibility in Handling Complex Systems**:

- Graph theoretical approaches are very easy to extend to more complex systems. For example, the traffic flow optimization could be used in a larger network including multiple intersections and roads.
- Non-linear equations, which can be difficult to solve analytically but local finesse and iterative graph type methods are able to accommodate various types of nonlinearities.

Limitations of Graph Theoretical Approaches:

1. Computational Complexity:

- Graph-based methods are a visual and intuitive way of providing explanations, but this approach can decrease clarity and result in increased computational complexity - for very large graphs or at the system scales with many variables.
- Such as on larger systems, using an adjacency matrix for representation and while performing Gaussian elimination has high computations costs.

2. Algorithmic Challenges:

- Implementing algorithms based on graphs for the solutions of nonlinear equations can be difficult. Point iteration for example, needs converge filled procedure and can suffer from the initial guess.
- Some optimization problems might contain nonlinear constraints, which lead to non-convex functions that are more difficult for the straight forward methods like Lagrange multipliers.

3. Limitations of Data Representation:

- These might miss certain problems in the context of other, it could be possible that a graph never reflects all aspects of some (say) hair raising issues. The graph model can oversimplify the issue in a case where relationships are difficult to represent as just edges or nodes.
- Score given to the corresponding approaches in a scale of strongly - disagrees until completely agrees Complex interactions when dealing with resource allocation problems could actually require more sophisticated graph structures or even hybrid methods

6.2. Potential Improvements

Areas for Further Research:

(i) Performance Enhancements for Large Graphs:Designing algorithms for performing research in large-scale graphs with hundreds or even thousands of variables and constraints. One might want to use techniques like representing the matrix as a sparse one, and parallel computing in ordered to cut down computational overhead.

(ii) Machine learning Integration: Machine learning techniques provide prediction and graphical algorithms deliver better optimization. As a simple example, combining graph-based methods with neural networks or reinforcement learning as done in recent research can allow the system to be more adaptive and intelligent. (Humplik et al., 2020)

(iii) Special Nonlinear Solvers:Further research is directed towards developing other graph-based solution methods for non-convex or very large scale complex constrained Nonlinear Equations. Finally, such graphs could top be further optimized using a combination of graph theory and more classical optimization techniques in hybrid approaches.

(iv) A Real-Time Application and Scalability:o Study the use of graph theoretical approaches to real-time systems which demand fast computation and scalability Real-time traffic management systems represent one-way new algorithms can be integrated, all of which help with handling changes in the network.

(v) Visualization Tools:Comprehensive Visualization: existing visualization methods for graph-based approaches, developers should focus on building new advanced tools in order to improve the understanding and usability These types of tools that give interactive and dynamic visualizations on large graphs, including their solutions (among other things) are useful for helping both practitioners and researchers to analyse the results.

Future Directions:

- Combining graph theoretical techniques with other mathematical and computational tools may yield stronger, more general solutions. As an example, integrating graph theory with optimization theories and computational geometry could lead to novel understandings and capabilities.
- It may also serve as a base where other domains/applications researchers can apply graph-based approaches (like bioinformatics, financial modelling or social network analysis) and develop new kind of research areas.

VII. Conclusion

7.1. Summary of Findings

Graph Theoretical Approaches:

(i) Linear Equations:We saw an example of linear equations given in the case study on optimization traffic flow and represented to be solved by Graph Theory. Optimal flow rates for the roads were obtained by modelling the traffic flow problem into a graph representation and solving this system with Gaussian elimination. This method clearly shows the figure and discipline that linear systems can obtain by graph theory.

(ii) Nonlinear Equations:o We have used graph theoretical techniques to solve the nonlinear equations with resource constraints and output functions as an example of Resource allocation. By using fixed-point iteration, we can deduce iterative solutions using graph structures so that closer SRA are achieved by a few steps to the optimal one. This illustrated that graph theory had the potential for dealing with solving complex nonlinear problems and simultaneously lower bounds on error were achievable as a byproduct of using pre-screens belonging to some embedding of treewidth.

Advantages Observed:

- ***Visualisation and Intuition***: Graph theory gives a graphical representation of the equations, which makes it very easy to understand how two constraints or relationships are being related for any given problem statement.
- ***Structured Methods***: The study demonstrates the organized nature of graph-based algorithms like Gaussian elimination and fixed-point iteration, which provides a distinct methodical path to addressing linear as well as non-linear equations.

Limitations Noted:

- ***Computational Complexity***:Big graph always leads to more computational challenges. However, addressing such systems will require efficient algorithms and optimizations.
- ***Algorithmic Challenges***: Fixed-point iteration and other graph-based methods can have issues with convergence and accuracy, especially in highly non-linear or complex situations.

7.2. Implications for Future Research

Potential Impact:

(i) Increased Computational Efficiency Enhanced: Future research should also be devoted towards more powerful graph analysis algorithms that deal with much larger graphs. Working with these large systems typically requires techniques designed to decrease the computational burden of graph theoretical calculations, such as those utilizing sparse matrix representations or parallel processing and approximate algorithms.

(ii) Integrations with Recent Technologies:Solid integration of graph theory with machine learning, AI and other advance computing can potentially make solutions more adaptive & intelligent Neural Networks + Graph theory: This could enhance predictive capabilities and optimization across several industries.

(iii) Real-World Applications:There are great potential advances that could follow from deploying graph theoretical methods in real-world problems in domains such as transportation, logistics, bioinformatics and finance. In this context, one of the questions that often comes up is whether a graph-based representation works-that can be enriched with practical implementations and case studies to evaluate if it truly does provide answers or solutions on large datasets.

***(iv) Advanced Graph-Based Methods*:**New methods using processed graphs to solve non-linear equations, especially if they contain complex constraints or are related with non-convex functions, have also become topics of great need in the research agenda. By combining graph theory with other optimization strategies, such hybrid methods can suggest new answers to deep issues.

***(v) Visualization and User Tools*:**Utilization of advanced visualization tools for graph theory development that will improve the way how these methods are understood and utilized. Interactive and real-time visualization tools can be useful for both researchers and practitioners when exploring a large number of graph-based solutions.

Future Directions:

- Investigating new and interdisciplinary fields for the application of graph theoretical methods, to create novel formulations for solutions that may be applicable. The reach of graph theory will only be able to grow further if it continues infiltrating into virgin territories and combines varied mathematical as well as computational approaches.

To sum up, Graph Theoretical Solutions provide efficient ways of solving linear as well non-linear equations with graphical representation and ordered way. This means that there is much work yet to be done in resolving challenges of computational efficiency and algorithmic complexity, but the opportunities for research applications are immense. There will be continued development in algorithms, interaction with contemporary technologies and the solving of real-life problems that depict a brighter future for the domain.

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Assistant Professor of Mathematics
Government College for Women, Chintamani, Karnataka, India
Email: mroopaprakash@gmail.com