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ON A LEFT TRUNCATED THREE PARAMETER LOGISTIC TYPE DISTRIBUTION WITH APPLICATIONS TO MANPOWER MODELING

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Abstract: Logistic distribution has gained a lot of momentum due to its applicability if analyzing several datasets arising at various places like Financial modeling, Manpower modeling, Biological experiments, and Agricultural experiments, etc. Several types of extensions have been given in the literature regarding logistic distribution. Among these, the three parameter logistic type distribution given by Srinivasa Rao et al. (2018) is more effective in modeling the phenomenon. Hence, in this paper, a Left Truncated Three Parameter Logistic Type Distribution is introduced, and various distributional properties such as distribution function, the four moments, skewness, kurtosis, hazard function, and survival function are derived. Some inferential aspects of the distribution are also studied. The utility of this distribution in analyzing the manpower situation in an organization is also presented.

Keywords

Left Truncated Distribution, Three Parameter Logistic Type Distribution, Distributional Properties, Manpower Modeling, Renewal Density, Promotional Policies.

1. Introduction

Berkson, Jack (1951) pointed out the use of logistic distribution after much work has been reported in literature regarding the expansion or generalization of logistic distribution, which is extensively used in modeling several datasets. Even though there is a close similarity in shape between the logistic and normal distribution the value of β_2 for logistic distribution is 4.2, which implies it is a leptokurtic distribution. Mustapha Muhammad et al. (2019) introduced a three-parameter probability model called Poisson generalized half logistic (PoiGHL). They discussed the relationship of PoiGHL with the exponentiated Weibull Poisson (EWP), Poisson exponentiated Erlang-truncated exponential (PEETE), and Poisson generalized Gompertz (PGG) model. Femi Samuel Adeyinka et al. (2019) studied the four parameters generalized log-logistic distribution using quadratic rank transmutation. Mohammad A. Aljarrah et al. (2020) considered a generalized asymmetric logistic distribution. The distribution consists of three parameters and was shown to fit a much wider range of heavy left and right tailed data. They developed some properties of distribution. Satheesh Kumar et al. (2022) have developed a wide class of generalized logistic distribution GGLD, which will be suitable for asymmetric data sets. They discussed some characteristics of the distribution. Abd-Elmonem A.M Teamah et al. (2024) presented a discrete distribution with one parameter derived by the discretization approach and called the discrete half-logistic distribution.

Indranil Ghosh et al. (2018) discussed the structural analysis of the EEL distribution and for illustrative purposes, a real-life data set was considered as an application of the EEL distribution. K.Srinivasa Rao et al. (2018) have introduced finite mixture of two parameter logistic type distribution and hierarchical clustering in analyzing image segmentation. This measure exhibits better than existing models. K. Srinivasa Rao and K.V. Satyanarayana et al. (2018) used two parameters logistic type distribution in image segmentation and image retrievals. It is observed that this algorithm outperforms the existing algorithm in segmenting for the images, which have platy kurtic distribution of pixel intensities. Realizing the importance of the logistic distribution in modeling the data for image analysis and retrieval, Srinivasa Rao et al. (2018), have introduced the three parameter logistic type distribution using the analogous arguments of Gram-Charlier series expansion and logistic distribution. In this study, they considered that the variable under consideration has an infinite range, i.e., the random variable can assume any value over the range $-\infty$ to $+\infty$, But in many practical situations, the random variables under study are constrained. For example, in manpower modeling, the complete length of service of an employee in any organization is non-negative, i.e., the lower limit of the variable is constrained to zero. Similarly, there exist several situations at places like inventory modeling, statistical quality control, and warranty data analysis where the variable under study is non-negative. This type of practical situation can be better analyzed by considering left truncated distribution. The majority of works reported in the literature regarding three logistic type distribution are not on a truncated nature. This motivated to develop and analyze the three parameter logistic type distribution which is useful for analyzing the datasets arising at manpower modeling, speech recognition, financial modeling, and quality control. Hence in this paper we introduce a new left truncated three parameter logistic type distribution.

The probability density function, distribution function, and other distributional properties such as moments, skewness, kurtosis, survival function, and hazard function are derived. The inferential aspects of the parameters are discussed using the likelihood function; the utility of the distribution in the analysis of the manpower situation is also studied. Two promotion policies, such as promotion by seniority and promotion by random, are also discussed.

2. Left Truncated Three Parameter Logistic Type Distribution

A continuous random variable X is said to have a left truncated three parameter logistic type distribution if its probability density function (p.d.f) is of the form

$$f(x;\mu,\sigma,s) = C\left[s+\left(\frac{x-\mu}{\sigma}\right)^2\right]\frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2} \qquad 0<x<\infty;\ \mu>0;\ \ \sigma>0;\ \ s>0 \tag{1}$$

The value of C is obtained such that $\int_{-\infty}^{\infty} f(x)dx = 1$

This implies, $C = \left[\left(s+\frac{\mu^2}{\sigma^2}\right)\frac{e^{\mu/\sigma}}{[1+e^{\mu/\sigma}]} - \frac{2\mu}{\sigma}\log\left(1+e^{\mu/\sigma}\right) - 2Li_2\left(-e^{\mu/\sigma}\right)\right]^{-1}$

where, $\mathrm{Li}_2(\cdot)$ is Dilogarithm Function

Here, μ is the location parameter, σ is the scale parameter and s is the drift parameter.

Making transformation $y = \frac{x-\mu}{\sigma}$ in the equation (1), we get

$$f(x;\mu,\sigma,s) = C\ [s+y^2]\frac{e^{-y}}{[1+e^{-y}]^2} \qquad 0<y<\infty; \tag{2}$$

Which may be called the standard left truncated three parameter logistic type distribution.

3. Distributional Properties

The various distributional properties of the left truncated three parameter logistic type distribution are discussed in this section. Different shapes of the frequency curves for given values of the parameters are shown in Figure 1

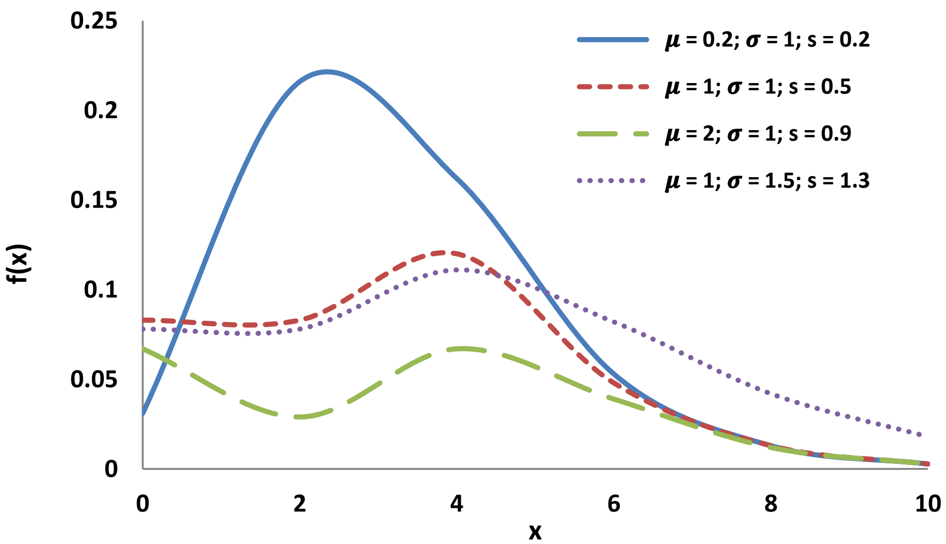

Figure 1: Left truncated three parameter logistic type distribution frequency curves for different values of parameters μ, σ and s.

The distribution function of the random variable X is

$$F_X(x) = \int_0^x f(t)dt$$

$$= C\int_0^x \left[s + \left(\frac{t-\mu}{\sigma}\right)^2\right] \frac{e^{-\left(\frac{t-\mu}{\sigma}\right)}}{\sigma\left[1 + e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]^2} dt$$

After simplifying, we get

$$F(x) = 1 - C\left\{\frac{\left[s + \left(\frac{x-\mu}{\sigma}\right)^2\right] e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\left[1 + e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]} + 2\left(\frac{x-\mu}{\sigma}\right)\log\left[1 + e^{-\left(\frac{x-\mu}{\sigma}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{x-\mu}{\sigma}\right)}\right)\right\} \tag{3}$$

where, C is as given in equation (1) and $Li_2(\cdot)$ is Dilogarithm Function

Mean of the distribution is

$$E(X) = \int_0^\infty x\, f(x)dx$$

$$= C\int_0^\infty x\left[s + \left(\frac{x-\mu}{\sigma}\right)^2\right] \frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1 + e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2} dx$$

After simplifying, we get

$$E(X) = \mu + C\sigma\left\{-\frac{\mu}{\sigma}\left[s + \frac{\mu^2}{\sigma^2}\right]\frac{e^{\mu/\sigma}}{[1 + e^{\mu/\sigma}]} + \left(s + \frac{3\mu^2}{\sigma^2}\right)\log\left(1 + e^{\mu/\sigma}\right) + \frac{6\mu}{\sigma}Li_2\left(-e^{\mu/\sigma}\right) - 6Li_3\left(-e^{\mu/\sigma}\right)\right\}$$

This implies

$$E(X) = C\left[\left(\frac{\mu^2 + s\sigma^2}{\sigma}\right)\log\left(1 + e^{\mu/\sigma}\right) + 4\mu Li_2\left(-e^{\mu/\sigma}\right) - 6\sigma Li_3\left(-e^{\mu/\sigma}\right)\right] \tag{4}$$

where, C is as given in equation (1) and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Second raw moment of the distribution is

$$\mu_2' = \int_0^\infty x^2\, f(x)\, dx$$

$$= C\int_0^\infty x^2\left[s + \left(\frac{x-\mu}{\sigma}\right)^2\right] \frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1 + e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2}\, dx$$

After simplifying, we get

$$\mu_2' = 2C\left[-(\mu^2 + s\sigma^2)Li_2\left(-e^{\mu/\sigma}\right) + 6Li_3\left(-e^{\mu/\sigma}\right) - 12\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right] \tag{5}$$

where, C is as given in equation (1) and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Third raw moment of the distribution is

$$\mu_3' = \int_0^\infty x^3\, f(x)\, dx$$

$$= C\int_0^\infty x^3\left[s + \left(\frac{x-\mu}{\sigma}\right)^2\right] \frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1 + e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2}\, dx$$

After simplifying, we get

$$\mu_3' = 6C\sigma\left[-(\mu^2 + s\sigma^2)Li_3\left(-e^{\mu/\sigma}\right) + 8\mu\sigma\, Li_4\left(-e^{\mu/\sigma}\right) - 20\sigma^2 Li_5\left(-e^{\mu/\sigma}\right)\right] \tag{6}$$

where, C is as given in equation (1) and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Fourth raw moment of the distribution is

$$\mu_4' = \int_0^\infty x^4 \, f(x)\, dx$$

$$= C\int_0^\infty x^4 \left[s + \left(\frac{x-\mu}{\sigma}\right)^2\right] \frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1 + e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2}\, dx$$

After simplifying, we get

$$\mu_4' = 24C\sigma^2\left[-(\mu^2 + s\sigma^2)Li_4\left(-e^{\mu/\sigma}\right) + 5\sigma(7-4\mu)Li_5\left(-e^{\mu/\sigma}\right) - 30\sigma^2 Li_6\left(-e^{\mu/\sigma}\right)\right] \tag{7}$$

where, C is as given in equation (1) and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

The Second central moment of the distribution is

$$\mu_2 = 2C\left[-(\mu^2 + s\sigma^2)Li_2\left(-e^{\mu/\sigma}\right) + 6Li_3\left(-e^{\mu/\sigma}\right) - 12\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right] - C^2K^2 \tag{8}$$

where, C is as given in equation (1)

$$K = \left[\left(\frac{\mu^2 + s\sigma^2}{\sigma}\right)\log\left(1 + e^{\mu/\sigma}\right) + 4\mu Li_2\left(-e^{\mu/\sigma}\right) - 6\sigma Li_3\left(-e^{\mu/\sigma}\right)\right]$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Third central moment of the distribution is

$$\begin{aligned}\mu_3 = {} & 6C\sigma\left[-(\mu^2 + s\sigma^2)Li_2\left(-e^{\mu/\sigma}\right) + 6Li_3\left(-e^{\mu/\sigma}\right) - 12\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right] \\ & - 2C^2K\left[\left(\frac{\mu^2 + s\sigma^2}{\sigma}\right)\log\left(1 + e^{\mu/\sigma}\right) + \left[\mu(4-3\mu) - 3\sigma^2\right]Li_2\left(-e^{\mu/\sigma}\right) + 6(3-\sigma)Li_3\left(-e^{\mu/\sigma}\right) - 36\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right]\end{aligned} \tag{9}$$

where, C is as given in equation (1)

$$K = \left[\left(\frac{\mu^2 + s\sigma^2}{\sigma}\right)\log\left(1 + e^{\mu/\sigma}\right) + 4\mu Li_2\left(-e^{\mu/\sigma}\right) - 6\sigma Li_3\left(-e^{\mu/\sigma}\right)\right]$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Fourth central moment of the distribution is

$$\begin{aligned}\mu_4 = {} & 24C\sigma^2\left[-(\mu^2 + s\sigma^2)Li_4\left(-e^{\mu/\sigma}\right) + 5\sigma(7-4\mu)\, Li_5\left(-e^{\mu/\sigma}\right) - 30\sigma^2 Li_6\left(-e^{\mu/\sigma}\right)\right] \\ & - 3C^2K\left\{8\sigma\left[-(\mu^2 + s\sigma^2)Li_3\left(-e^{\mu/\sigma}\right) + 8\mu\sigma Li_4\left(-e^{\mu/\sigma}\right) - 20\sigma^2 Li_5\left(-e^{\mu/\sigma}\right)\right]\right. \\ & \left. - 4CK\left[-(\mu^2 + s\sigma^2)Li_2\left(-e^{\mu/\sigma}\right) + 6Li_3\left(-e^{\mu/\sigma}\right) - 12\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right] + C^2K^3\right\}\end{aligned} \tag{10}$$

where, C is as given in equation (1)

$$K = \left[\left(\frac{\mu^2 + s\sigma^2}{\sigma}\right)\log\left(1 + e^{\mu/\sigma}\right) + 4\mu Li_2\left(-e^{\mu/\sigma}\right) - 6\sigma Li_3\left(-e^{\mu/\sigma}\right)\right]$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

The skewness of the distribution is

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_1 = \frac{\begin{array}{c}\left[6C\sigma\left[-(\mu^2 + s\sigma^2)Li_2\left(-e^{\mu/\sigma}\right) + 6Li_3\left(-e^{\mu/\sigma}\right)\right] - 12\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right] - 2C^2K\left[\left(\frac{\mu^2+s\sigma^2}{\sigma}\right)\log\right. \\ \left.\left(1 + e^{\mu/\sigma}\right) + \left[\mu(4-3\mu) - 3\sigma^2\right]Li_2\left(-e^{\mu/\sigma}\right) + 6(3-\sigma)Li_3\left(-e^{\mu/\sigma}\right) - 36\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right]\Big]^2\end{array}}{\left[2C\left[-(\mu^2 + s\sigma^2)Li_2\left(-e^{\mu/\sigma}\right) + 6Li_3\left(-e^{\mu/\sigma}\right) - 12\sigma^2 Li_4\left(-e^{\mu/\sigma}\right)\right] - C^2K^2\right]^3} \tag{11}$$

where, C is as given in equation (1)

$$K = \left[\left(\frac{\mu^2 + s\sigma^2}{\sigma}\right)\log\left(1 + e^{\mu/\sigma}\right) + 4\mu Li_2\left(-e^{\mu/\sigma}\right) - 6\sigma Li_3\left(-e^{\mu/\sigma}\right)\right]$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n (n ≥ 2)

The kurtosis of the distribution is

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 = \frac{\begin{matrix}[24C\sigma^2[-(\mu^2+s\sigma^2)Li_4(-e^{\mu/\sigma})+5\sigma(7-4\mu)Li_5(-e^{\mu/\sigma})-30\sigma^2Li_6(-e^{\mu/\sigma})] \\ -3C^2K\{8\sigma[-(\mu^2+s\sigma^2)Li_3(-e^{\mu/\sigma})+8\mu\sigma Li_4(-e^{\mu/\sigma})-20\sigma^2Li_5(-e^{\mu/\sigma})] \\ -4CK[-(\mu^2+s\sigma^2)Li_2(-e^{\mu/\sigma})+6Li_3(-e^{\mu/\sigma})-12\sigma^2Li_4(-e^{\mu/\sigma})]+C^2K^3\}]\end{matrix}}{[2C[-(\mu^2+s\sigma^2)Li_2(-e^{\mu/\sigma})+6Li_3(-e^{\mu/\sigma})-12\sigma^2Li_4(-e^{\mu/\sigma})]-C^2K^2]^2} \tag{12}$$

where, C is as given in equation (1)

$$K = \left[\left(\frac{\mu^2+s\sigma^2}{\sigma}\right)\log\left(1+e^{\mu/\sigma}\right)+4\mu Li_2\left(-e^{\mu/\sigma}\right)-6\sigma Li_3\left(-e^{\mu/\sigma}\right)\right]$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n (n ≥ 2)

4. Estimation of Parameters

In this section we consider the estimation of the parameters of the left truncated three parameter logistic type distribution.

Method of Moments

According to this method, the moments of the population and the sample are equated correspondingly to deduce the estimators of the parameters. Assume that we have a sample of size 'n' drawn from a population having the probability density function of the form given by

$$f(x;\mu,\sigma,s) = C\left[s+\left(\frac{x-\mu}{\sigma}\right)^2\right]\frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2} \qquad 0<x<\infty;\ \mu>0;\ \sigma>0;\ s>0$$

where, C is as given in equation (1) and $Li_2(\cdot)$ is Dilogarithm Function

This distribution having three parameters μ, σ^2 and s. Hence, we consider the first two raw moments of the sample and population, which leads to the following equation.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = C\left\{\left(\frac{\mu^2+s\sigma^2}{\sigma}\right)\log\left(1+e^{\mu/\sigma}\right)+4\mu Li_2\left(-e^{\mu/\sigma}\right)-6\sigma Li_3\left(-e^{\mu/\sigma}\right)\right\} \tag{13}$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = 2C\left[-(\mu^2+s\sigma^2)Li_2\left(-e^{\mu/\sigma}\right)+6Li_3\left(-e^{\mu/\sigma}\right)-12\sigma^2Li_4\left(-e^{\mu/\sigma}\right)\right] \tag{14}$$

where, C is as given in equation (1) and $Li_n(\cdot)$ is Polylogarithm Function of order n (n ≥ 2)

Solving the equations (13) and (14) simultaneously, we get the moment estimates of μ, σ^2 and s.

Maximum Likelihood Method of Estimation

Let $x_1, x_2, x_3, \ldots, x_n$ be a sample of size n drawn sample from a population having the probability density function of the form of given in equation (1). Then the likelihood function of the sample is

$$L = C^n\sigma^{-n}\prod_{i=1}^{n}\left\{\left[s+\left(\frac{x_i-\mu}{\sigma}\right)^2\right]\frac{e^{-\left(\frac{x_i-\mu}{\sigma}\right)}}{\left[1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}\right]^2}\right\} \tag{15}$$

where, C is as given in equation (1) and $Li_2(\cdot)$ is Dilogarithm Function

The log likelihood function of the sample is

$$\log L = -n\log\sigma - n\log\left[\left(s+\frac{\mu^2}{\sigma^2}\right)\frac{e^{\mu/\sigma}}{[1+e^{\mu/\sigma}]} - \frac{2\mu}{\sigma}\log\left(1+e^{\mu/\sigma}\right) - 2\,Li_2\left(-e^{\mu/\sigma}\right)\right] + \sum_{i=1}^{n}\log\left[s+\left(\frac{x_i-\mu}{\sigma}\right)^2\right] - \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right) - 2\sum_{i=1}^{n}\log\left[1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}\right] \tag{16}$$

For obtaining the maximum likelihood estimators of the parameters we have to maximize L or Log L with respect to the parameters μ, σ^2 and s.

$$nC\left\{\left(s+\frac{\mu^2}{\sigma^2}\right)\frac{e^{\mu/\sigma}}{[1+e^{\mu/\sigma}]^2} - 2\left(1+\frac{\sigma}{e^{\mu/\sigma}}\right)\log\left(1+e^{\mu/\sigma}\right)\right\} + 2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu}{\sigma}\right)}{\left[s+\left(\frac{x_i-\mu}{\sigma}\right)^2\right]} + 2\sum_{i=1}^{n}\frac{1}{\left[1+e^{\left(\frac{x_i-\mu}{\sigma}\right)}\right]} = n \tag{17}$$

$$-nC\left\{-\frac{\mu}{\sigma}\left(s+\frac{\mu^2}{\sigma^2}\right)\frac{e^{\mu/\sigma}}{[1+e^{\mu/\sigma}]^2} - \frac{2\mu}{\sigma^2}\left(\frac{1}{\sigma^2}-\mu\right)\frac{e^{\mu/\sigma}}{[1+e^{\mu/\sigma}]} + 2\left(\frac{\mu}{\sigma}-\frac{\sigma}{e^{\mu/\sigma}}\right)\log\left(1+e^{\mu/\sigma}\right)\right\} + \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right) - 2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu}{\sigma}\right)^2}{\left[s+\left(\frac{x_i-\mu}{\sigma}\right)^2\right]} - 2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu}{\sigma}\right)}{\left[1+e^{\left(\frac{x_i-\mu}{\sigma}\right)}\right]} = n \tag{18}$$

$$-nC\left(\frac{e^{\mu/\sigma}}{[1+e^{\mu/\sigma}]}\right) + \sum_{i=1}^{n}\frac{1}{\left[s+\left(\frac{x_i-\mu}{\sigma}\right)^2\right]} = 0 \tag{19}$$

where, C is as given in equation (1) and $Li_2(\cdot)$ is Dilogarithm Function

Solving the equations (17), (18) and (19) simultaneously using numerical methods like Newton Rapson method, we will get the parameter estimates of $\hat{\mu}$, $\hat{\sigma}^2$ and $\hat{s}$.

5. Manpower Planning Model with Left Truncated Three Parameter Logistic Type Distribution

In this section, the manpower model as the complete length of service distribution (CLS) of an organization using a left truncated three parameter logistic type distribution. Let the complete length of service of an employee in an organization follows a half logistic type distribution with parameters μ, σ^2 and s. Its probability density function, distribution function and expected length of service are given in the equations (1), (3) and (4) respectively.

The probability that an individual survives in the organization for the length of time 't' is G(t), which is the survival function of the employee in the organization. It can be defined as the complement of the distribution function F(t). Then

$$G(t) = 1 - F(t)$$

$$G(t) = C\int_t^{\infty}\left[s+\left(\frac{x-\mu}{\sigma}\right)^2\right]\frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2}\,dx$$

Making necessary transformations and by integration, we get

$$G(t) = C\left\{\frac{\left[s+\left(\frac{t-\mu}{\sigma}\right)^2\right]e^{-\left(\frac{t-\mu}{\sigma}\right)}}{\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]} + 2\left(\frac{t-\mu}{\sigma}\right)\log\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-\mu}{\sigma}\right)}\right)\right\} \tag{20}$$

where, C is as given in Equation (1) and $Li_2(\cdot)$ is Dilogarithm Function

Different shapes of survival function G(t) for given values of the parameters are shown in

Figure 2, 3 and 4.

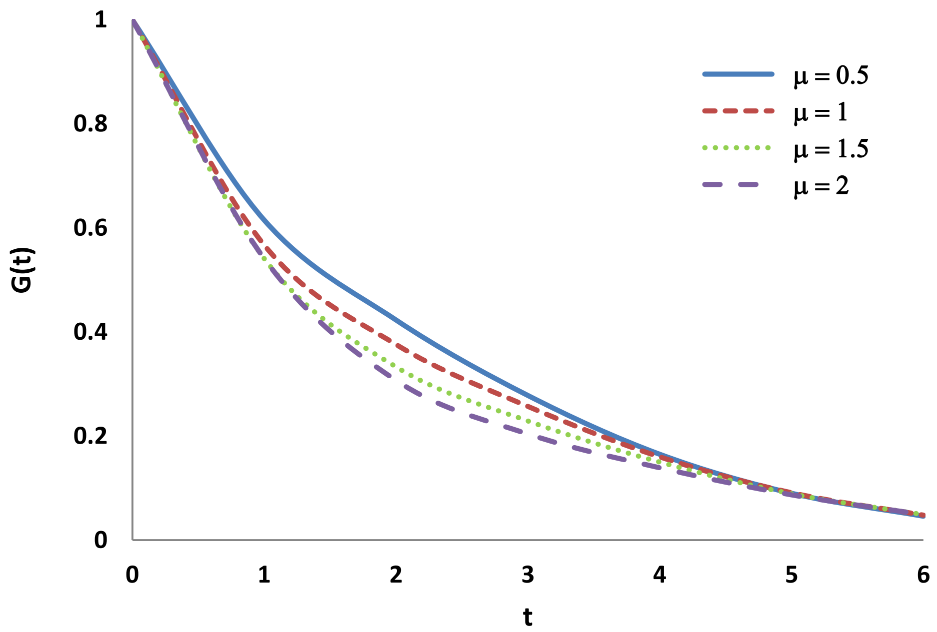

Figure 2: Shapes of the Survivor Function for different values of parameter μ (σ= 1 and s = 2)

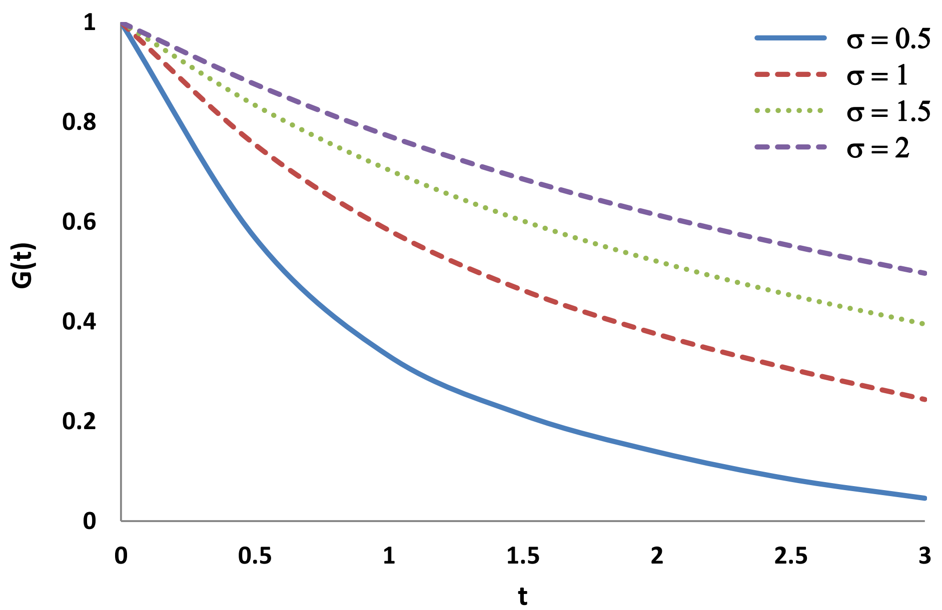

Figure 3: Shapes of the Survivor Function for different values of parameter σ (μ = 1 and s = 3)

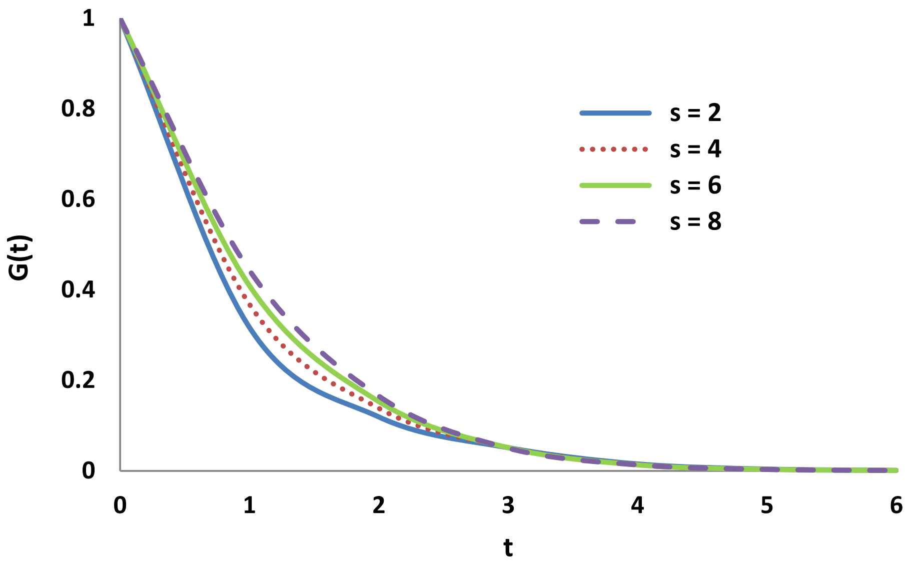

Figure 4: Shapes of the Survivor Function for different values of parameter s (μ = 1.5 and σ = 0.5)

The force of separation, also known as loss of intensity or rate of labour wastage, can be obtained as

$$L(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{G(t)}$$

Substituting the functional form of f(t) and G(t) from the equations (1) and (20), we get

$$L(t) = \frac{\left[s + \left(\frac{t-\mu}{\sigma}\right)^2\right] e^{-\left(\frac{t-\mu}{\sigma}\right)}}{\sigma\left[1 + e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]^2 \left\{\frac{\left[s+\left(\frac{t-\mu}{\sigma}\right)^2\right] e^{-\left(\frac{t-\mu}{\sigma}\right)}}{\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]} + 2\left(\frac{t-\mu}{\sigma}\right)\log\left[1 + e^{-\left(\frac{t-\mu}{\sigma}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-\mu}{\sigma}\right)}\right)\right\}} \tag{21}$$

where, $Li_2(\cdot)$ is Dilogarithm Function

For different values of parameters the shapes of labour wastage L(t) is shown in Figures 5, 6 and 7

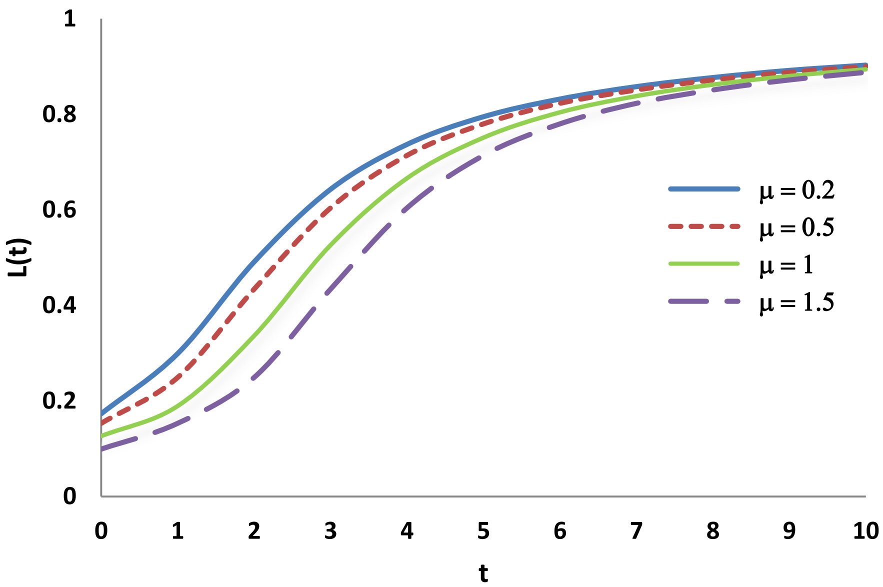

Figure 5: Loss of Intensity curve for different values of parameter μ (σ = 1 and s = 2)

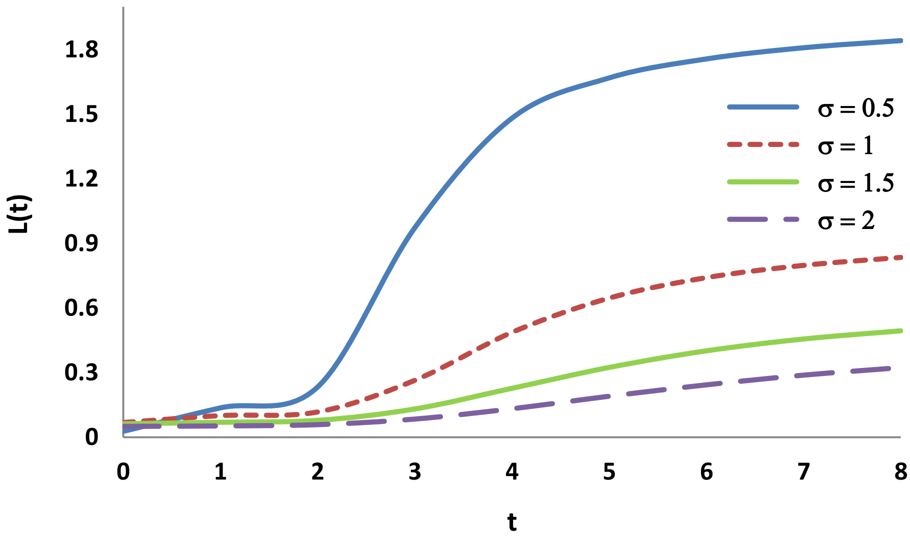

Figure 6: Loss of Intensity curve for different values of parameter σ (μ = 2 and s = 1)

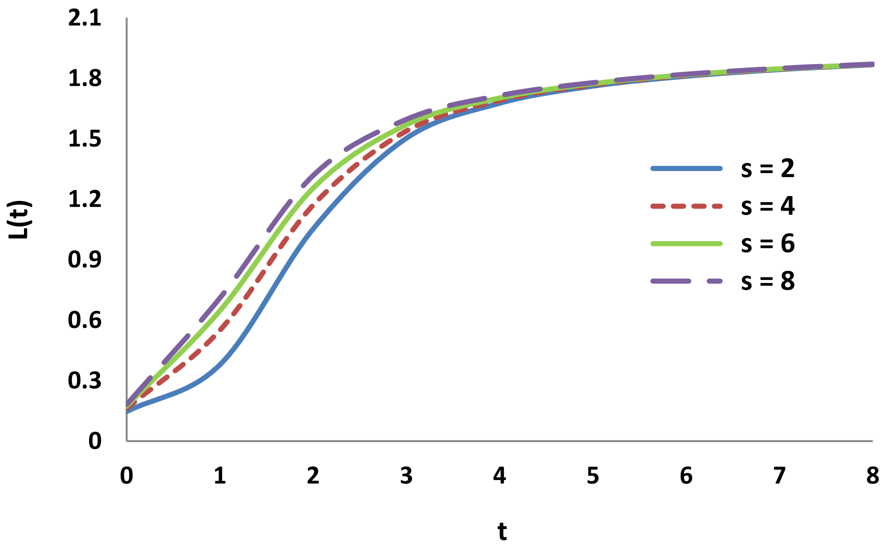

Figure 7: Loss of Intensity curve for different values of parameter s (μ = 1 and σ = 0.5)

The renewal density of this model is

$$h(t) = f(t) + \int_0^t G(t-x)\, f(x) dx$$

Using the values of f(t) and G(t) from the equation (1) and (20) and after simplification one can get

$$h(t) = C\left[s + \left(\frac{t-\mu}{\sigma}\right)^2\right] \frac{e^{-\left(\frac{t-\mu}{\sigma}\right)}}{\sigma\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]^2} + C^2 \int_0^t \left[s + \left(\frac{x-\mu}{\sigma}\right)^2\right] \frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2} \left\{ \frac{\left[s + \left(\frac{t-x-\mu}{\sigma}\right)^2\right] e^{-\left(\frac{t-x-\mu}{\sigma}\right)}}{\left[1+e^{-\left(\frac{t-x-\mu}{\sigma}\right)}\right]} + 2\left(\frac{t-x-\mu}{\sigma}\right) \log\left[1+e^{-\left(\frac{t-x-\mu}{\sigma}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-x-\mu}{\sigma}\right)}\right) \right\} dx \quad (22)$$

where, C is as given in equation (1) and $Li_2(\cdot)$ is Dilogarithm Function

For different values of parameters the shape of renewal density function h(t) is shown in Figures 8, 9 and 10

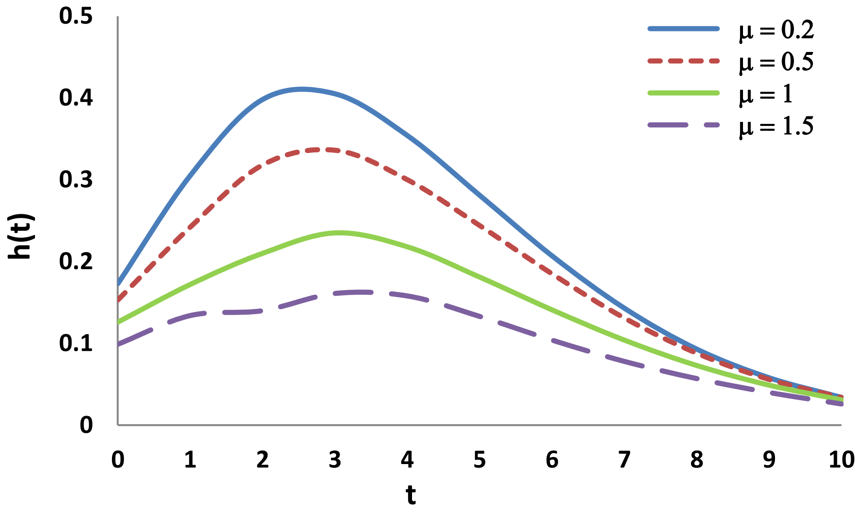

Figure 8: Shapes of Renewal density function h(t) for different values of parameter μ (σ= 1 and s = 2)

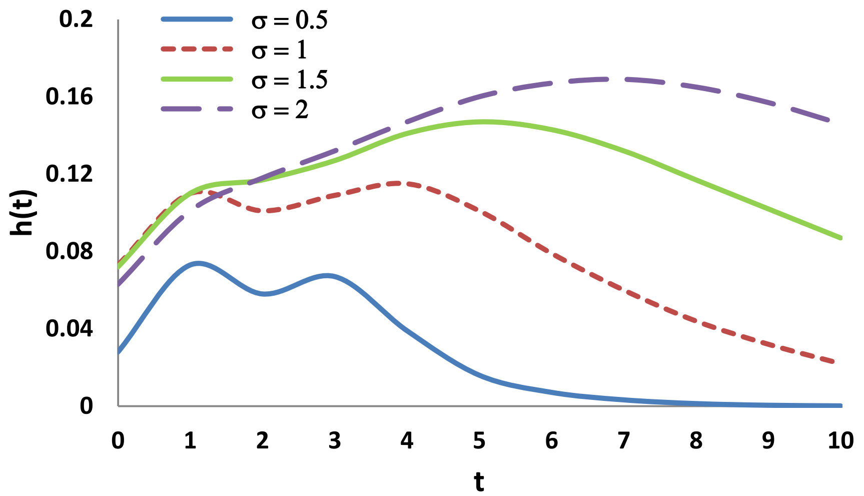

Figure 9: Shapes of Renewal density function h(t) for different values of parameter σ (μ= 2 and s = 2)

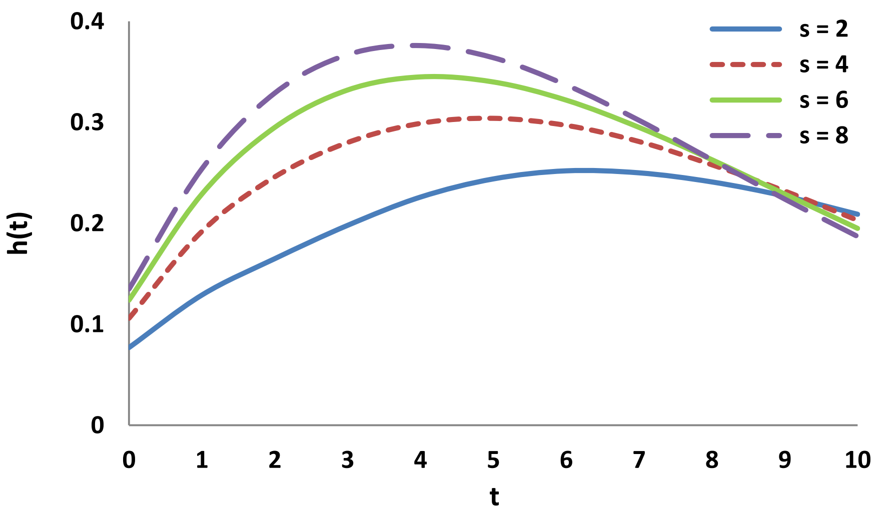

Figure 10: Shapes of Renewal density function h(t) for different values of parameter s (μ = 1 and σ = 2)

6. Manpower Planning Model with Left Truncated Three Parameter Logistic Type Distribution having CLS Distribution

In the previous section, it is assumed that the complete length of service distribution (CLS) is same for the entire period of time. However, in some organizations, the employee behavior changes after a spending some period of time in the organization, and these changes may be resulting out of change in the behavior of management due to changes in the economic situation, education factors, recruitment policies, modernization/globalization effects, etc., So, the change incomplete length of service distribution (**CLS**) should be considered for an effective estimation of the number of persons leaving the organization and, hence, accurate estimates for the number of recruiters. In this case, the complete length of service distribution (**CLS**) of an employee in the organization before and after the change are assumed as left truncated three parameter logistic distribution with parameter μ_1, σ_1^2, s_1 and μ_2, σ_2^2, s_2 respectively.

Let the probability density function of the complete length of service (**CLS**) for a person who joins the organization at time "X" after it was established be $f_X(t)$. It is assumed that this probability density has the same functional form for all "X", but its parameters are changing depending on the state of the organization. i.e., they have the same functional form of "X".

Let

$$f_1(t) = C_1\left[s_1 + \left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]\frac{e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\sigma_1\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]^2} \qquad 0 < t < \infty;\ \ \mu_1 > 0;\ \ \sigma_1 > 0;\ s_1 > 0 \tag{23}$$

where $C_1 = \left[\left(s_1 + \frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]} - \frac{2\mu_1}{\sigma_1}\log\left(1+e^{\mu_1/\sigma_1}\right) - 2Li_2\left(-e^{\mu_1/\sigma_1}\right)\right]^{-1}$

and $\mathrm{Li}_2(\cdot)$ is Dilogarithm Function be the probability density function of the complete length of service distribution (**CLS**) for members recruited before the change and

Let

$$f_2(t) = C_2\left[s_2 + \left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]\frac{e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\sigma_2\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]^2} \qquad 0 < \ t < \infty;\ \ \mu_2 > 0;\ \ \sigma_2 > 0;\ s_2 > 0 \tag{24}$$

where, $C_2 = \left[\left(s_2 + \frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]} - \frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right) - 2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right]^{-1}$

and $\mathrm{Li}_2(\cdot)$ is Dilogarithm Function be the probability density function of the complete length of service distribution (**CLS**) for members recruited after the change in the organization.

The distribution functions of $F_1(t)$ and $F_2(t)$ are

$$F_1(t) = 1 - C_1\left\{\frac{\left[s_1 + \left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]} + 2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+\ e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right] - \ 2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\} \tag{25}$$

where, $C_1 = \left[\left(s_1 + \frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]} - \frac{2\mu_1}{\sigma_1}\log\left(1+e^{\mu_1/\sigma_1}\right) - 2Li_2\left(-e^{\mu_1/\sigma_1}\right)\right]^{-1}$

and $\mathrm{Li}_2(\cdot)$ is Dilogarithm Function

$$F_2(t) = 1 - C_2\left\{\frac{\left[s_2 + \left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]} + 2\left(\frac{t-\mu_2}{\sigma_2}\right)\log\left[1+\ e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\} \tag{26}$$

where, $C_2 = \left[\left(s_2 + \frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]} - \frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right) - 2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right]^{-1}$

and $\mathrm{Li}_2(\cdot)$ is Dilogarithm Function

The Survival functions before and after changes are

$$G_1(t) = C_1\left\{\frac{\left[s_1 + \left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]} + 2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\} \tag{27}$$

where, $C_1 = \left[\left(s_1 + \frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]} - \frac{2\mu_1}{\sigma_1}\log\left(1+e^{\mu_1/\sigma_1}\right) - 2Li_2\left(-e^{\mu_1/\sigma_1}\right)\right]^{-1}$

and $Li_2(\cdot)$ is Dilogarithm Function

and

$$G_2(t) = C_2\left\{\frac{\left[s_2 + \left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]} + 2\left(\frac{t-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\} \tag{28}$$

where, $C_2 = \left[\left(s_2 + \frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]} - \frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right) - 2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right]^{-1}$

and $Li_2(\cdot)$ is Dilogarithm Function

7. The Renewal Density of the Model with Changing CLS

Let $d_1(t)$ be the density function of the residual complete length of service distribution (**CLS**). This is the distribution of the remaining length of service of a member of the system selected at random at time zero. As called by Cox (1962), the process for 't' greater than or equal to zero is known as the modified renewal process. The renewal density satisfies the integral equation

$$h(t) = d_1(t) + \int_0^t h(t-x)\, f_2(x)dx \tag{29}$$

On taking Laplace transform on each side of equation (29), we get

$$h^*(s) = \frac{d_1^*(s)}{[1-f_2^*(s)]} \tag{30}$$

Its approximation is given by Bartholomew (1973) as

$$h^o(t) = d_1(t) + \frac{D_1(t)F_2(t)}{\int_0^t G_2(x)dx} \tag{31}$$

where,

$$d_1(t) = \frac{G_1(t)}{m_1} \tag{32}$$

and

$$D_1(t) = \int_0^t G(x)\, h(t-x)dx$$

$$D_1(t) = \int_0^t d_1(x)dx \tag{33}$$

m_1 is the average length of the complete length of service (**CLS**) before change. For the model under study, density function of the residual complete length of service distribution (**CLS**) is

$$d_1(t) = \frac{G_1(t)}{m_1}$$

On simplification,

$$d_1(t) = \frac{1}{K_1}\left\{\frac{\left[s_1 + \left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]} + 2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\} \tag{34}$$

where, $K_1 = \left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\left(1+e^{\mu_1/\sigma_1}\right) + 4\mu_1 Li_2\left(-e^{\mu_1/\sigma_1}\right) - 6\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)\right]$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Consider

$$D_1(t) = \int_0^t d_1(x)\, dx$$

$$D_1(t)=\int_0^t \frac{1}{K_1}\left\{\frac{\left[s_1+\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right]}+2\left(\frac{x-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right]-2Li_2\left(-e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right)\right\}dx$$

On simplification, we get

$$D_1(t)=1+\frac{\sigma_1}{K_1}\left\{-\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]+2\left(\frac{t-\mu_1}{\sigma_1}\right)\left[1+\left(\frac{t-\mu_1}{\sigma_1}\right)\right]Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)+6Li_3\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\} \tag{35}$$

where, $K_1=\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\left(1+e^{\mu_1/\sigma_1}\right)+4\mu_1 Li_2\left(-e^{\mu_1/\sigma_1}\right)-6\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)\right]$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Consider

$$\int_0^t G_2(x)\,dx=C_2\int_0^t\left\{\frac{\left[s_2+\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right]}+2\left(\frac{x-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right]-2Li_2\left(-e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right)\right\}dx$$

On simplification,

$$=C_2\sigma_2\left\{-\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]+2\left(\frac{t-\mu_2}{\sigma_2}\right)\left[1+\left(\frac{t-\mu_2}{\sigma_2}\right)\right]Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)+6Li_3\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\}+C_2K_2 \tag{36}$$

where, $C_2=\left[\left(s_2+\frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]}-\frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right)-2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right]^{-1}$

$$K_2=\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log\left(1+e^{\mu_2/\sigma_2}\right)+4\mu_2 Li_2\left(-e^{\mu_2/\sigma_2}\right)-6\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right)\right]$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

Therefore

$$h^o(t)=d_1(t)+\frac{D_1(t)\,F_2(t)}{\int_0^t G_2(x)\,dx}$$

Substituting equations (34), (35), (36), and (26), we get

$$h^o(t)=\frac{1}{K_1}\left\{\frac{\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]}+2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\}$$

$$+\frac{\left\{1+\frac{\sigma_1}{K_1}\left\{-\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]+2\left(\frac{t-\mu_1}{\sigma_1}\right)\left[1+\left(\frac{t-\mu_1}{\sigma_1}\right)\right]Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)+6Li_3\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\}\right\}F_2(t)}{\left\{C_2\sigma_2\left\{-\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]+2\left(\frac{t-\mu_2}{\sigma_2}\right)\left[1+\left(\frac{t-\mu_2}{\sigma_2}\right)\right]Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)+6Li_3\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\}\right\}+C_2K_2}$$

where,

$$F_2(t)=1-C_2\left\{\frac{\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]}+2\left(\frac{t-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\}$$

$$K_1=\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\left(1+e^{\mu_1/\sigma_1}\right)+4\mu_1 Li_2\left(-e^{\mu_1/\sigma_1}\right)-6\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)\right]$$

$$K_2=\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log\left(1+e^{\mu_2/\sigma_2}\right)+4\mu_2 Li_2\left(-e^{\mu_2/\sigma_2}\right)-6\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right)\right]$$

$$C_2=\left[\left(s_2+\frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]}-\frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right)-2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right]^{-1}$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

8. Recruitment Policies for Manpower Models with Left Truncated Three Parameter Logistic Type Distribution

In the previous section, the survival behavior rate of labour wastage and renewal rates are studied when the complete length of service (CLS) follows the left truncated three parameter logistic type distribution in normal complete length of service (CLS) and in the case of changing complete length of service (CLS). In this section, discussions are made on the different recruitment policies of an organization, which play a vital role in operating system strategies. An organization can be viewed as having two grades, the first grade is training grade (Grade I) and the second grade is organization grade (Grade II). Here, the training grade was introduced to provide experienced men to fill variances, which arise within an organization grade. The organization itself will, of course, in general possesses a hierarchical structures. However, the movement between the different grades within an organization grade were considered. The whole organization is considered as single unit.

Let N_1 and N_2 denote the size of the training grade and the organization grades respectively, and further $N= N_1+N_2$. Here, N_2 is assumed to be known and fixed. The problem under discussion is in finding the size of the training grade in order that, on the average, people spend some fixed period of time 't' in it before promotion into the organization grade.

In this study, there are two rules namely i) Promotion by seniority i.e. the most senior (and hence most experienced) member of the training grade is promoted and ii) Promotion by random with respect to the length of service .i.e., promotion is made on some other basis, like ability, personal qualities, qualifications, special trainings, etc., are examined.

Let W_1 be the individual loss rate in training grade (Grade I), W_2 is the individual loss rate in organization grade (Grade II), and P is the promotion rate.

Since the input and output of each grade must balance, for grade II, it can be written as

$$N_1 P = N_2 W_2 \tag{37}$$

Moreover, in equilibrium the expected input to the system per unit time is $\frac{N}{m}$, where m is the mean length of completed service. Thus for Grade I,

$$\frac{N}{m} = N_1(P + W_1)$$

$$= N_1 W_1 + N_2 W_2 \tag{38}$$

In gender W_1, W_2 and P are functions of time. It is shown by Bartholomew (1973) that P, W_1, W_2 tend to equilibrium values which are independent of the age of the system.

Let m_1 be the average time spent in Grade I. Then, in equilibrium, the expected number of variances occurring per unit time in this grade is

$$\frac{N_1}{m_1} = N_1(P + W_1)$$

This implies

$$\frac{1}{m_1} = P + W_1 \tag{39}$$

From Equations (38) and (39)

$$\frac{N}{m} = \frac{N_1}{m_1} \tag{40}$$

This equation states that the through–put for the whole system is equal to the number of variances in Grade I

9. Promotion by Seniority when the Complete Length of Service Follows Left Truncated Three Parameter Logistic Type Distribution

Let $a(t/_T)$ denote the age distribution of the system at time T, given that the system was established at T=0. Thus $a(t/_T)\ \delta t$ is the probability that an individual chosen at random 'T' has a length of service in $(t, t + \delta t)$.

Thus

$$a\left(\frac{t}{T}\right)\delta t = \text{Prob \{Individual joined in } (T-t, T-t+\delta t) \text{ and remained for time } t\}$$

$$= h(T - t)\ \delta t\ G(t)$$

where, h(t) is the renewal density of the whole system

Hence

$$a\left(\frac{t}{T}\right) = h(T - t)\delta t\ G(t)$$

This holds for $t < T$ we have,

$$\lim_{T\to\infty} h(T) = \frac{1}{m}$$

Now

$$\lim_{T\to\infty} G(T) = 0$$

and

$$\lim_{T\to\infty} a\left(\frac{t}{T}\right) = \frac{G(T)}{m}$$

$$= a(t)$$

Which is the equilibrium age distribution.

Since a loss from Grade II is replaced by the most senior member of Grade I, it follows that at anytime every individual in Grade II has a length of service at least as long as any individual in Grade I, and hence there exists some threshold value at t_1, such that all individuals with a length of service less than that t_1 are in Grade I. 't_1' is a random variable, but if the grade sizes are larger than the approximate formula for its expected value is

$$\int_{t_1}^{\infty} a(t)\ dt = \frac{N_2}{N_1 + N_2} \tag{41}$$

It follows that in equilibrium the expected number of promotions per unit time will be the promotion of new recruits who service to the threshold length of service of t_1.

i.e., t_1

$$N_1 P = G(t_1)\frac{N}{m} \tag{42}$$

Let m_L be the average length of time spent in Grade I by those who leave while still in the Grade I, and let m_P be the average length of time spent in Grade I by those who are eventually promoted to Grade II. Then consider the problem of choosing N_1 so that m_P has some predetermined value. In this case, m_P is equivalent to the average value of introduced earlier, and hence equations (41) and (42) hold.

Thus one can have

$$\int_{m_P}^{\infty} a(t)\ dt = \frac{N_2}{N}$$

i.e.,

$$\frac{1}{m}\int_{m_P}^{\infty} G(t)\ dt = \frac{N_2}{N} \tag{43}$$

and

$$N_1 P = G(m_P)\frac{N}{m}$$

equation (43) gives $R = \frac{N_1}{N_2}$ as a function of m_P and hence, knowing the size N of the organization, one can determine N_1 for specified value of m_P.

This implies

$$\frac{1}{m}\int_{m_P}^{\infty} G(t)\ dt = \frac{1}{1 + R} \tag{44}$$

The complete length of service of an employee in the organization is assumed to follow a left truncated three parameter logistic type distribution. Substituting an equation (20) in (44), one can get

$$\frac{C}{m}\int_{m_P}^{\infty}\left\{\frac{\left[s+\left(\frac{t-\mu}{\sigma}\right)^2\right]e^{-\left(\frac{t-\mu}{\sigma}\right)}}{\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]}+2\left(\frac{t-\mu}{\sigma}\right)\log\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu}{\sigma}\right)}\right)\right\}dt=\frac{1}{1+R}$$

where, C is as given in equation (1)

and $Li_2(\cdot)$ is Dilogarithm Function

Making necessary transformations and by integration, we get

$$\frac{C\sigma}{m}\left\{\left[s+\left(\frac{m_P-\mu}{\sigma}\right)^2\right]\log\left[1+e^{-\left(\frac{m_P-\mu}{\sigma}\right)}\right]-4\left(\frac{m_P-\mu}{\sigma}\right)Li_2\left(-e^{-\left(\frac{m_P-\mu}{\sigma}\right)}\right)-6Li_3\left(-e^{-\left(\frac{m_P-\mu}{\sigma}\right)}\right)\right\}=\frac{1}{1+R}$$

m_P is the average length of time spent in Grade I by those who are eventually promoted to Grade II.

On Simplification,

$$R=\frac{m\left\{\left(s+\frac{\mu^2}{\sigma^2}\right)\frac{e^{\mu/\sigma}}{[1+e^{\mu/\sigma}]}-\frac{2\mu}{\sigma}\log\left(1+e^{\mu/\sigma}\right)-2Li_2\left(-e^{\mu/\sigma}\right)\right\}}{\sigma\left\{\left[s+\left(\frac{m_P-\mu}{\sigma}\right)^2\right]\log\left[1+e^{-\left(\frac{m_P-\mu}{\sigma}\right)}\right]-4\left(\frac{m_P-\mu}{\sigma}\right)Li_2\left(-e^{-\left(\frac{m_P-\mu}{\sigma}\right)}\right)-6Li_3\left(-e^{-\left(\frac{m_P-\mu}{\sigma}\right)}\right)\right\}}-1 \qquad (45)$$

where, $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

For various values of m, m_P, μ, σ, and s corresponding values of R are computed using equation (45) and presented in Table 1.

From Table 1, it is observed that as σ increases, the value of R is decreasing. As μ increases the value of R is increasing and s increases the value of R is increasing and for m_P increases the R value is increasing. An increase in m, results increase in R.

Table 1

m	m_P	μ	σ	s	R
10	4	2	1	3	41.542
10	4	2	1.5	3	12.044
10	4	2	2	3	5.721
10	4	2	2.5	3	3.326
10	4	3	1.5	3	12.325
10	4	3.5	1.5	3	12.651
10	4	4	1.5	3	13.138
10	4	4.5	1.5	3	13.845
10	4	2	1.5	4	13.192
10	4	2	1.5	6	15.295
10	4	2	1.5	8	17.176
10	4	2	1.5	10	18.868
10	5	2	1.5	3	18.52
10	6	2	1.5	3	29.33
10	7	2	1.5	3	47.46
10	8	2	1.5	3	78.082
20	4	2	1.5	3	25.088
30	4	2	1.5	3	38.132
40	4	2	1.5	3	51.176
50	4	2	1.5	3	64.221

10. Promotion by Random when the Complete Length of Service Follows Left Truncated Three Parameter Logistic Type Distribution

Let $F_1(t)$ be the probability that an individual remains in the system for a time 't' without being promoted and let n_t be the number of promotions in $(0,t)$. Then we have,

$F_1(t) = \text{Prob \{Individual not promoted in } (0,t)\text{/does not leave in } (0,t)\} * \text{Prob \{does not leave in } (0,t)\}$

$F_1(t) = \left(1 - \frac{1}{N_1}\right)^{n_t} G(T)$

The excepted value of n_t is N_1P_t

Thus as a first approximation one can take

$$F_1(t) = G(t)\left[1 - \frac{1}{N_1}\right]^{N_1P_t}$$

$N_1 \to \infty$ gives

$$F_1(t) = G(t)\, e^{-P_t}$$

Then the average time spent in Grade I is

$$m_1 = \int_0^\infty F_1(t)\, dt = \int_0^\infty G(t)\, e^{-P_t}\, dt$$

But from equation (40)

$$\frac{m_1}{m} = \frac{N_1}{N}$$

Thus

$$\frac{1}{m}\int_0^\infty G(t)e^{-P_t}\, dt = \frac{N_1}{N} \tag{46}$$

In equilibrium, let q(t) be the probability density function of the time interval between entry of an individual into Grade I and his promotion to Grade II, given that he is promoted before leaving.

Let $Q(t) = \int_t^\infty q(x)\, dx$

Since $Q(t) = \text{prob \{Not promoted in a period of length t / doesnot leave in that period\}}$

$$= \left(1 - \frac{1}{N_1}\right)^{n_t}$$

where, n_t is the number of promotions in a interval of length t.

Replacing n_t can be replaced by its expected value, N_1P_t. We get the

$$Q(t) = \left(1 - \frac{1}{N_1}\right)^{N_1P_t}$$

Therefore

$$m_p = \int_0^\infty Q(t)\, dt = \int_0^\infty e^{-P_t}\, dt = \frac{1}{P} \tag{47}$$

Using this in conjunction with equation (46) gives N_1 in terms of m_P, and hence

$$\frac{N_1}{N} = \frac{1}{m}\int_0^\infty G(t)\, e^{-t/m_P}\, dt \tag{48}$$

The expression for the value "R" is computed from the following equation.

$$\frac{1}{m}\int_0^\infty G(t)\, e^{-t/m_P}\, dt = \frac{R}{R+1} \tag{49}$$

Substituting G(t) as given in the equation (20), we get

$$\frac{C}{m}\int_0^\infty e^{-t/m_P}\left\{\frac{\left[s+\left(\frac{t-\mu}{\sigma}\right)^2\right]e^{-\left(\frac{t-\mu}{\sigma}\right)}}{\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]}+2\left(\frac{t-\mu}{\sigma}\right)\log\left[1+e^{-\left(\frac{t-\mu}{\sigma}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu}{\sigma}\right)}\right)\right\}dt=\frac{R}{1+R} \tag{50}$$

where, C is as given in equation (1) and $Li_2(\cdot)$ is Dilogarithm Function

By solving the equation (50), for various values of m, m_P, μ, σ and s the R values are computed and presented in Table 2.

Table 2

m	m_p	μ	σ	s	R
7	4	0.5	1	2	0.2504
7	4	0.5	1.5	2	0.3516
7	4	0.5	2	2	0.4361
7	4	0.5	2.5	2	0.5072
5	2	1	0.5	4	0.1501
5	2	1.5	0.5	4	0.1562
5	2	2	0.5	4	0.1681
5	2	2.5	0.5	4	0.1829
5	2	1	2	2	0.3704
5	2	1	2	4	0.3667
5	2	1	2	6	0.3646
5	2	1	2	8	0.3633
5	1.5	0.5	1	2	0.2137
5	2	0.5	1	2	0.2614
5	2.5	0.5	1	2	0.3014
5	3	0.5	1	2	0.3353
8	4	0.5	1	2	0.2125
9	4	0.5	1	2	0.1845
10	4	0.5	1	2	0.1630
11	4	0.5	1	2	0.1461

From Table 2, it is observed that the recruit policy is significantly increased by complete length of service distribution (CLS) parameters. As the shape parameter σ increases the value of R is increasing contrary to the policy of promotion by seniority when other parameters remaining fixed. It is also observed that as μ increases the value of R is increasing and also observed that as s increases the value of R is decreasing, this decrease is very small. When m_P is increasing the value of R increasing. It is also observed that m increases the value of R is decreasing.

Conclusion

In this paper, we introduced a new and novel Left Truncated Three Parameter Logistic Type Distribution. The three parameter logistic type distribution has lot of utility in analyzing several data sets as an alternative to the Gaussian distribution were the variable under study is leptokurtic. The various distributional properties such as distribution function, Hazard function, Survival function are derived. It is observed that the survival function decreases gradually as the location parameter μ increases. An application of the proposed distribution in analyzing manpower situation at an organization is presented by considering the complete length of service of an employee in the organization follows a Left Truncated Three Parameter Logistic Type Distribution. The renewal density function and loss density function are derived and analyzed. Two promotion policies such as promotion by seniority and promotion by random are discussed. It is observed that promotion by random is more effective in reducing the iteration of employees in the organization. The proposed manpower model is much useful for HR managers in performing HR analytics effectively. This distribution can be further extended to the case of the Doubly Truncated Three Parameter Logistic Type Distribution which will be taken up elsewhere.

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