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A NOTE ON GENERIC CONTROLLABILITY OF NETWORKS WITH IDENTICAL SISO DYNAMICAL NODES

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Abstract. Controllability of interconnected linear systems have been an area of interest for the past few decades as it has applications in numerous fields of science and technology. Classical theory of control deals with systems with known parameters. In practical applications it may not be possible to estimate exact parameter values. Hence the available numerical conditions like Kalman's rank condition, PBH criteria etc. for controllability are hard to verify. To address this difficulty, properties that are preserved for all but few values of parameters, i.e., generic properties must be considered. In [9], Commault et al. has provided a necessary and sufficient condition for the generic controllability of a homogeneous networked system. In this paper, we have given an example to show that this result may not be true in general. Another example is provided to show that one of the conditions in this result is not necessary. Also, we have obtained some necessary conditions for the generic controllability of a heterogeneous networked system. The obtained results are substantiated with examples.

1. Introduction

During the past few decades, an increase in interest is observed in controllability studies. The notion of controllability was introduced by R.E. Kalman in the second half of the $20th$ century[17, 18] which measured the ability of a dynamical system to get to a desired final state from an arbitrary initial state in a finite time. It dealt with single higher dimensional systems with known parameter values. In practical applications, as the parameter values governing the system properties may vary or never be known precisely, a more comprehensive framework for controllability, called structural controllability was established by C.T. Lin[19]. The notion of structural controllability emphasized the important role possessed by the network structure in the controllability of dynamical systems. Numerous studies have been done with regard to many kinds of systems to understand these concepts in detail over the past few decades and various conditions have been obtained. Over the course, it became evident that the modelling of real life systems required complex networks[5, 28, 31].

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The studies of individual systems connected together can be traced back to the work done by Gilbert [13] which was then followed by many others as the controllability and observability of interconnected systems became a topic of interest [6, 8, 10, 12]. For large scale networks, it is almost impossible to obtain the exact parameter values describing the dynamics of the system. Structural controllability was introduced to overcome the difficulties caused by this fact. Following the seminal work by Lin in 1974, the conditions obtained for single input systems were enhanced to multi input systems and studied in detail by Glover et al.[14] as well as Shields et al. [27]. Later, the proof of the structural controllability theorem was simplified by Linnemann[20]. The idea of strong structural controllability, which is the controllability of the system for any values of the indetrminate parameter values of the system was also introduced and some necessary and sufficient conditions were given by Mayeda et al.[23]. Hosoe et al.[16] modified the algebraic condition for structural controllability using the irreducibility condition on the adjoined system matrices. A graph theoretic interpretation of the condition that the structured matrix obtained by adjoining the state matrix and control matrix having full rank was obtained by Mayeda[22]. In [9], Commault et al. tries to study a new notion of controllability of a system, named generic controllability where the system matrices are fixed for each nodes but the links between nodes have unknown weights. This is a relatively new concept in the area of controllability of inter connected systems. However, the conditions obtained are structural as they are based on the composition of the network graph. As the applications of complex networks in various fields of science and technology increased, studies on controllability of networks of dynamical systems also increased. New tools were introduced to study the structural controllability of networks and is still an active area of research [4, 7, 21, 25, 24, 34]. The time-line of research in the area of structural systems can be traced through the studies of Dion et al.[11], Ramos et al.[26] and Xiang et al.[32].

Along with the studies on structural controllability of networked systems, numerous conditions for state controllability of interconnected systems were obtained in this time period[1, 3, 2, 15, 29, 30, 33]. Commault et al. [9] examined the generic controllability of inter connected systems where individual systems having same dynamics are connected together and obtained a necessary and sufficient condition for generic controllability of networked systems. In this paper, we have proved that these conditions are necessary for the generic controllability of networked systems having heterogeneous dynamics. The obtained results are substantiated with examples. The sections are arranged as follows: Preliminaries in Section 2 are followed by formulation of the controllability problem in section 3. Some necessary conditions for generic controllability of networked systems is obtained in Section 4. The obtained results are illustrated with examples. Concluding remarks and future works are stated in section 5.

2. Notations and Preliminaries

The field of real numbers is denoted by $\mathbb{R}$ and the linear space containing n-vectors of real numbers is denoted by $\mathbb{R}^n$. I represents the identity matrix of appropriate order, the $n \times 1$ vector with 1 at the i^{th} position and rest all zeros is denoted by e_i and the set of all $m \times n$ matrices with real entries is denoted by $\mathbb{R}^{m\times n}$. Consider the following system

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) \tag{2.1}$$

in which $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ denotes the input vector. $\tilde{A} \in \mathbb{R}^{n\times n}, \tilde{B} \in \mathbb{R}^{n\times m}$ denotes the state matrix and the control matrix of the system (2.1). A directed graph, $G = (V, E)$ can be correlated with (2.1). $V = V_1 \cup V_2$ represents vertex set, where V_1 is the state vertex set and V_2 is the control vertex set. A path in graph G, from vertex v_o to v_m is a chain of edges $(v_0, v_1), (v_1, v_2), \dots (v_{m-1}, v_m)$, where $v_0, v_1, \dots, v_m \in V$ and $(v_{i-1}, v_i) \in E$, for $i = 1, 2, \dots, m$. If the initial vertex of a path belong to the V_2 and the end vertex belong to the V_1, then the path is called *control-state path.* A *stem* is a control-state path which does not pass through the same vertex twice. A system in which every state vertex is the end vertex of a control state path is called *control connected.*

3. Problem Formulation

Consider a networked system, with N state nodes and m control nodes interacting via weighted directed connections. The weighted directed graph $G(\mathcal{N}) = (V_{\mathcal{N}}, E_{\mathcal{N}})$, called the network graph can be used to represent the network, $\mathcal{N}$. The vertex set of the network graph is given by, $V_{\mathcal{N}} = \{v_1, v_2, \dots, v_N\} \cup \{u_1, u_2, \dots, u_m\}$, where $v_i's$ and $u_i's$ represent the state nodes and control nodes respectively. The directed connections between the nodes is represented by the edge set $E_{\mathcal{N}}$. Edge weights assigned to the network graph quantifies the strength of the communication between the individual nodes.

The node v_i represents a dynamical system with n states, a scalar input w_i, and a scalar output y_i. The dynamics of the node v_i is given by

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i w_i(t) \\ y_i(t) &= C_i x_i(t) \end{aligned} \tag{3.1}$$

where $A_i \in \mathbb{R}^{n\times n}$ for each i and B_i(respectively, C_i) is a $n-$ dimensional column vector (respectively, a row vector) for each i. The dynamic state of each node is defined by the matrices (A_i, B_i, C_i).

Combining the state space model representing the dynamics of each node with the composition of the network graph, we get a global system $\sum_{\mathcal{N}}$ of state space dimension Nn and m control inputs. The input signal for the node i is given by the weighted combination of control signals in line with the network graph

$$w_i(t) = \sum_{i=1}^{N} \gamma_{ij} y_j(t) + \sum_{l=1}^{m} \delta_{il} u_l(t) \tag{3.2}$$

where γ_{ij} represents the connection strength of the link from node v_j to node v_i, δ_{il} represents the connection strength of the link from control node u_l to the state node v_i. γ_{ij} and δ_{il} becomes zero when there is no edge in the network graph between the state nodes or from a control node to a state node respectively. Let $\Gamma = [\gamma_{ij}]_{N\times N}, i = 1, 2, \dots, N, j = 1, 2, \dots, N$ and $\Delta = [\delta_{il}]_{N\times m}, i = 1, 2, \dots, N, l = 1, 2, \dots, m$ represent network topology.

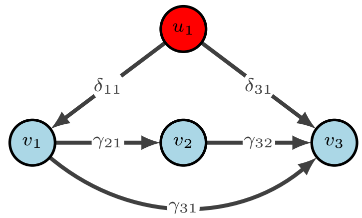

FIGURE 1. Network graph with $\Gamma = \begin{bmatrix} 0 & 0 & 0 \\ \gamma_{21} & 0 & 0 \\ \gamma_{31} & \gamma_{32} & 0 \end{bmatrix}$ and $\Delta = \begin{bmatrix} \delta_{11} \\ 0 \\ \delta_{31} \end{bmatrix}$.

Then the compact form of $\sum_{\mathcal{N}}$ is given by

$$\sum_{\mathcal{N}} : \dot{x}(t) = Fx(t) + Gu(t) \tag{3.3}$$

where $x(t) = (x_1(t), \ldots, x_m(t))^T$ and $u(t) = (u_1(t), \ldots, u_m(t))^T$, with $(.)^T$ indicates the transpose of a matrix. The matrices F and G representing the state and control matrices of $\sum_{\mathcal{N}}$ respectively have dimensions $Nn \times Nn$ and $Nn \times m$. They are of the following form:

$$F = \begin{bmatrix} A_1 + \gamma_{11}B_1C_1 & \gamma_{12}B_1C_2 & \ldots & \gamma_{1N}B_1C_N \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1}B_NC_1 & \gamma_{N2}B_NC_2 & \ldots & A_N + \gamma_{NN}B_NC_N \end{bmatrix}$$

and

$$G = \begin{bmatrix} \delta_{11}B_1 & \delta_{12}B_1 & \ldots & \delta_{1m}B_1 \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{N1}B_N & \delta_{N2}B_N & \ldots & \delta_{Nm}B_N \end{bmatrix}$$

In this work, our aim is to analyse the controllability of $\sum_{\mathcal{N}}$, using the dynamics of the individual systems. i.e., Using $(A_i, B_i, C_i)'s$, and structure of the networked system. The matrices $(A_i, B_i, C_i)'s$ are assumed to be exact and known, but the network communication strength are not fixed precisely. i.e., we know whether the entries are zero or non-zero, but does not know the exact parameter values.

4. Main Results

In [9], Commault et al. give the following set of conditions which are necessary and sufficient for the generic controllability of interconnected systems with identical dynamical nodes.

Theorem 4.1. [9] *Consider a network $\mathcal{N}$ with N internal nodes, m control nodes with $N > m$, and its graph $G(\mathcal{N})$. Assume that all nodes are identical, SISO, n^{th}-order dynamical systems defined by matrices A, B, C. The global system $\Sigma_{\mathcal{N}}$ is generically controllable if and only if the following conditions hold:*

- (i) *The pair (A, B) is controllable.*
- (ii) *The pair (C, A) is observable.*

(iii) The graph $G(\mathcal{N})$ is control-connected.
(iv) The internal nodes of $G(\mathcal{N})$ can be covered by a disjoint set of stems and cycles.

We will prove that the first three conditions in Theorem 4.1 are necessary for the generic controllability of networked systems with non-identical nodes.

Theorem 4.2. *If the pair (A_i, B_i) is not controllable for some i, say i_0, then the global system is not generic controllable.*

Proof: Suppose that (A_i, B_i) is not controllable for some i, say i_0 , then by PBH criterion there exists a scalar λ and a row vector v such that $v\left(A_{i_0} - \lambda I\right) = 0$ and $vB_{i_0} = 0$. Now consider the vector $e_{i_0} \otimes v$, where $e_{i_0} \in \mathbb{R}^{1\times N}$ with i_0^{th} entry 1 and all other entries zero. Then

$$\left(e_{i_0} \otimes v\right)(F - \lambda I) = 0 \quad and \quad \left(e_{i_0} \otimes v\right) G = 0$$

i.e., (F, G) is not controllable.

Example 4.3. Consider a networked system with state matrices $A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and control matrices $B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The output matrices are given by $C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Now for $\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$ and $\Delta = \begin{bmatrix} \delta_{11} \\ \delta_{21} \end{bmatrix}$ we get $F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1+\gamma_{11} & \gamma_{12} & 0 \\ 0 & 0 & 1 & 1 \\ 0 & \gamma_{21} & \gamma_{22} & 1 \end{bmatrix}$ and $G = \begin{bmatrix} 0 \\ \delta_{11} \\ 0 \\ \delta_{12} \end{bmatrix}$. Here (A_1, B_1) is controllable but (A_2, B_2) is not controllable. Therefore by Theorem4.2 the given system is not controllable. By PBH criteria, we can verify this, as the matrix $[F - I, G]$ has rank at-most 3 only.

Theorem 4.4. *If $N > m$, for the global system to be generic controllable atleast one of the pairs $(A_i, C_i), i = 1, 2, \ldots, N$ must be observable.*

Proof: Suppose that (A_i, C_i) is not observable for all $i = 1, 2, \ldots, N$. Then by PBH criteria there exists a scalar λ and a column vector v_i such that $(A_i - \lambda I) v_i = 0$ and $C_i v_i = 0$. Now consider the vector $(e_i \otimes v_i)$, where $e_i \in \mathbb{R}^{1\times N}$ with ith entry 1 and all other entries zero. Then

$$(F - \lambda I)(e_i \otimes v_i) = 0$$

Then $rank\,[F - \lambda I, G] \leq N(n-1)+m < Nn$. Therefore (F, G) is not controllable.

Example 4.5. Consider a networked system with state matrices $A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and control matrices $B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The output matrices are given by $C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$. Now for $\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$ and $\Delta = \begin{bmatrix} \delta_{11} \\ \delta_{21} \end{bmatrix}$ we get

$F = \begin{bmatrix} 1+\gamma_{11} & 0 & 0 & \gamma_{12} \\ 1+\gamma_{11} & 1 & 0 & \gamma_{12} \\ 0 & 0 & 1 & 1 \\ \gamma_{21} & 0 & 0 & 1+\gamma_{22} \end{bmatrix}$ and $G = \begin{bmatrix} \delta_{11} \\ \delta_{11} \\ 0 \\ \delta_{12} \end{bmatrix}$. Here both (A_1, C_1) and (A_2, C_2) are not observable. Also $N > m$. Therefore by Theorem4.4 the given system is not controllable. By PBH criteria, we can verify this, as the matrix $[F - I, G]$ has rank at-most 3 only.

Theorem 4.6. *If the graph $G(\mathcal{N})$ is not control connected, then the global system is not generic controllable.*

Proof: Suppose that $G(\mathcal{N})$ is not control connected. Rearrange the nodes so that the first k nodes represent the non control connected nodes. Then the matrices Γ and Δ can be expressed as $\Gamma = \begin{bmatrix} \Gamma_{11} & 0 \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix}$ and $\Delta = \begin{bmatrix} 0_{k\times k} \\ \Delta_2 \end{bmatrix}$ where Γ_{11} is a $k \times k$ matrix. Then F and G are of the form $F = \begin{bmatrix} F_{11} & 0 \\ F_{21} & F_{22} \end{bmatrix}$ and $G = \begin{bmatrix} 0_{kn\times kn} \\ G_2 \end{bmatrix}$ where F_{11} is a $kn \times kn$ matrix. Now for any left eigenvector v of F_{11}, $\tilde{v} = \begin{bmatrix} v & 0_{n(N-k)} \end{bmatrix}$ is a left eigenvector of F with $vG = 0$. Therefore (F, G) is not controllable.

Example 4.7. Consider a networked system with state matrices $A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and control matrices $B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The output matrices are given by $C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Here both (A_1, C_1) and (A_2, C_2) are observable. Also (A_1, B_1) ans (A_2, B_2) are controllable. Now let $\Gamma = \begin{bmatrix} 0 & \gamma_{12} \\ 0 & 0 \end{bmatrix}$ and $\Delta = \begin{bmatrix} \delta_{11} \\ 0 \end{bmatrix}$.

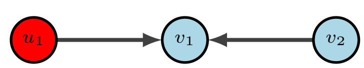

FIGURE 2. Clearly, $G(\mathcal{N})$ is not control connected.

Then by Theorem4.6 the given system is not controllable. For, we have $F = \begin{bmatrix} 1 & 0 & \gamma_{12} & 0 \\ 1 & 1 & \gamma_{12} & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $G = \begin{bmatrix} \delta_{11} \\ \delta_{11} \\ 0 \\ 0 \end{bmatrix}$. By PBH criteria, we can verify that the given system is not controllable, as the matrix $[F - I, G]$ has rank at-most 3 only.

5. Conclusion and Future works

The generic controllability of interconnected linear systems with heterogeneous dynamics is studied. It has been shown that some of the necessary conditions for the generic controllability of homogeneous networked systems stay necessary for heterogeneous networked systems also and the obtained results are supplemented with suitable examples. In this article, individual systems are considered as single-input single-output systems. Hereafter we intend to analyse the generic controllability of networked systems where the individual nodes are multi-input multi-output systems.

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